

# Solution

## MATH 133, QUIZ #7

- (1) Which of the following infinite series are geometric series? If you encounter a geometric series, determine whether it diverges or converges and find its value. (6 points):

(a)

$$\sum_{n=1}^{\infty} \frac{n}{2^n}$$

Not a geometric Series

(b)

$$\begin{aligned} \sum_{n=1}^{\infty} (-1)^{n-1} 2^n &= \sum_{n=1}^{\infty} \frac{(-1)^n}{-1} (2)^n \\ &= \sum_{n=1}^{\infty} -1 \cdot (-2)^n \end{aligned}$$

geometric Series w/  
ratio  $r = -2$ ,  $|r| > 1$ .

Thus, it diverges

(c)

$$\sum_{n=2}^{\infty} (-1)^n \frac{2^{n-1}}{3^n} = \sum_{n=2}^{\infty} \left(\frac{-2}{3}\right)^n \cdot \frac{1}{2}$$

geometric Series w/  $r = -\frac{2}{3}$ ,  $|r| < 1$ ; thus it converges.

its sum is  $S = \frac{1^{\text{st}} \text{ term}}{1-r} = \frac{\frac{1}{2} \left(-\frac{2}{3}\right)^2}{1 - \left(-\frac{2}{3}\right)} = \boxed{\frac{2}{15}}$

- (2) Write the number  $0.\overline{123} = 0.123123123\dots$  as a quotient of two integers. Show your work (2 points).

$$0.\overline{123} = \frac{123}{10^3} + \frac{123}{10^6} + \frac{123}{10^9} + \dots$$

$$= \sum_{n=1}^{\infty} 123 \left(\frac{1}{10^3}\right)^n \quad \text{geometric Series w/ } r = \frac{1}{10^3}$$

Its Sum is  $\frac{1^{\text{st term}}}{1-r} = \frac{\frac{123}{1000}}{1 - \frac{1}{1000}} = \boxed{\frac{123}{999}}$

- (3) Does the following infinite series converge or not? (2 points)

$$\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$$

Let  $f(x) = \frac{1}{x \ln(x)}$  for  $x \in [2, \infty)$ . Then  $f$  is positive, continuous ( $x \geq 2$ ) and decreasing (denominator increases with  $x$ ) on  $[2, \infty)$ .

$$\text{Now, } \int_2^{\infty} f(x) dx = \lim_{t \rightarrow \infty} \int_2^t \frac{1}{x \ln x} dx = \lim_{t \rightarrow \infty} (\ln|\ln(x)|) \Big|_2^t$$

$$= \lim_{t \rightarrow \infty} (\ln|lnt| - \ln|\ln(2)|) = \boxed{\infty}.$$

Since  $\int_2^{\infty} f(x) dx$  diverges, then by the integral test,

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n} \quad \boxed{\text{diverges as well.}}$$