

Solution

1) Evaluate the integral

$$\int \frac{x+4}{x^2+6x+5} dx$$

$$\frac{x+4}{x^2+6x+5} = \frac{x+4}{(x+5)(x+1)} = \frac{A}{x+5} + \frac{B}{x+1}; \quad \text{So}$$

$$x+4 = A(x+1) + B(x+5)$$

$$\left. \begin{array}{l} x=-1 \quad 3 = 4B \Rightarrow B = \frac{3}{4} \\ x=-5 \quad -1 = -4A \Rightarrow A = \frac{1}{4} \end{array} \right\}$$

$$\begin{aligned} & \int \frac{\frac{1}{4}}{x+5} + \frac{\frac{3}{4}}{x+1} dx \\ &= \boxed{\frac{1}{4} \ln|x+5| + \frac{3}{4} \ln|x+1| + C} \end{aligned}$$

2) Evaluate the integral

first, we compute $\int_e^\infty \frac{1}{x(\ln x)^3} dx = \int u^{-3} du$ where $u = \ln x$
 $du = \frac{1}{x} dx$

$$= \int u^{-3} du = \frac{u^{-2}}{-2} + C = -\frac{1}{2 \ln^2 x} + C.$$

$$\begin{aligned} \text{Now, } \int_e^\infty \frac{1}{x \ln^3 x} dx &= \lim_{t \rightarrow \infty} \int_e^t \frac{1}{x \ln^3 x} dx \\ &= \lim_{t \rightarrow \infty} \left(\frac{-1}{2 \ln^2(x)} \Big|_e^t \right) \\ &= \lim_{t \rightarrow \infty} \left(\frac{-1}{2 \ln^2(t)} + \frac{1}{2 \ln^2(e)} \right) = -\frac{1}{\infty} + \frac{1}{2 \cdot 1} = \boxed{\frac{1}{2}} \end{aligned}$$