

3pts

1) Evaluate the integral

$$\int (x+2)e^{2x} dx$$

Let $u = x+2$, $dv = e^{2x} dx$. Then $du = dx$, $v = \frac{1}{2}e^{2x}$. Then,

$$\int u dv = uv - \int v du \quad (+1)$$

$$= (x+2) \cdot \frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x} dx \quad (+1)$$

$$= \frac{x+2}{2}e^{2x} - \frac{1}{4}e^{2x} + C \quad \leftarrow (+1)$$

$$= e^{2x} \left(\frac{x}{2} + \frac{3}{4} \right) + C = \frac{e^{2x}}{4} (2x+3) + C$$

3pts

2) Evaluate the integral

$$\int_0^{\pi} 3 \cos^3 2x dx$$

$$\int_0^{\pi} 3 \cos^3(2x) dx = 3 \int_0^{\pi} \cos^2(2x) \cos(2x) dx$$

$$= 3 \int_0^{\pi} (1 - \sin^2(2x)) \cos(2x) dx \quad (+1)$$

Let $u = \sin(2x)$, $\frac{du}{2} = \cos(2x) dx$. Then

$$\int_0^{\pi} 3 \cos^3(2x) dx = \frac{3}{2} \int_0^0 (1-u^2) du = 0.$$

(+1)

4pts

3) Evaluate the integral

(+1)

$$\int \frac{x^2}{\sqrt{4-x^2}} dx$$

Let $x = 2\sin\theta$, $dx = 2\cos\theta d\theta$. Then

$$\begin{aligned} \int \frac{x^2 dx}{\sqrt{4-x^2}} &= \int \frac{4\sin^2\theta \cdot 2\cos\theta d\theta}{\sqrt{4\cos^2\theta}} = \frac{4 \cdot 2}{2} \int \frac{\sin^2\theta \cos\theta d\theta}{\cos\theta} \\ &= 4 \int \sin^2\theta d\theta \quad \leftarrow (+1) \\ &= \frac{4}{2} \int 1 - \cos(2\theta) d\theta \\ &= 2 \left(\theta - \frac{1}{2} \sin(2\theta) \right) + C \quad \leftarrow (+1) \\ &= 2\theta - \sin(2\theta) + C \\ &= 2\theta - 2\sin(\theta)\cos(\theta) + C \\ &= 2\arcsin\left(\frac{x}{2}\right) - 2 \cdot \frac{x}{2} \cdot \sqrt{1 - \left(\frac{x}{2}\right)^2} + C \quad \leftarrow (+1) \\ &= 2\arcsin\left(\frac{x}{2}\right) - \frac{x}{2} \sqrt{4-x^2} + C. \end{aligned}$$

Some useful formulas:

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\begin{aligned} \cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 1 - 2\sin^2\theta \\ &= 2\cos^2\theta - 1 \end{aligned}$$