

# Solution

## Math 133, Quiz #5

1. Compute the integral

$$\int x^2 \ln x \, dx.$$

Let  $u = \ln x$ ,  $dv = x^2 \, dx$ ; Then

$du = \frac{1}{x} \, dx$ ,  $v = \frac{x^3}{3}$ . By integrating by parts, we have

$$\begin{aligned} \int x^2 \ln x \, dx &= uv - \int v \, du = \frac{x^3}{3} \ln(x) - \int \frac{x^3}{3} \cdot \frac{1}{x} \, dx = \frac{x^3}{3} - \frac{1}{3} \int x^2 \, dx \\ &= \frac{x^3}{3} \ln x - \frac{1}{3} \frac{x^3}{3} + C \\ &= \boxed{\frac{x^3}{3} \ln x - \frac{x^3}{9} + C} \end{aligned}$$

2. Compute the following limit if it exists

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\ln^2 x}{x^{1/3}} \quad (\text{form } \frac{\infty}{\infty}) &\stackrel{(L.H.)}{=} \lim_{x \rightarrow \infty} \frac{\frac{2 \ln x \cdot \frac{1}{x}}{\frac{1}{3} x^{-2/3}}}{\frac{1}{3} x^{-2/3}} = \lim_{x \rightarrow \infty} \frac{6 \ln x}{x^{1/3}} \quad \left( \frac{\infty}{\infty} \right) \\ &\stackrel{(L.H.)}{=} \lim_{x \rightarrow \infty} \frac{\frac{6}{x}}{\frac{1}{3} x^{-2/3}} = \lim_{x \rightarrow \infty} \frac{18}{x^{1/3}} = \frac{18}{\infty} = \boxed{0} \end{aligned}$$

3. Write the functions  $\sinh x$  and  $\cosh x$  in terms of the exponential function.  
Then evaluate  $\cosh^2 x - \sinh^2 x$ .

By definition,  $\cosh x = \frac{1}{2}(e^x + e^{-x})$ ,  $\sinh x = \frac{1}{2}(e^x - e^{-x})$ . So,

$$\cosh^2 x - \sinh^2 x = \frac{1}{4}(e^{2x} + 2e^x e^{-x} + e^{-2x}) - \frac{1}{4}(e^{2x} - 2e^x e^{-x} + e^{-2x})$$

$$= \frac{1}{4}[2e^0 + 2e^0] = \frac{1}{4}[4] = \boxed{1}$$