

Solution

1) Evaluate the integral

$$\int \frac{dx}{\sqrt{x}(1+x)}$$

Hint: Use substitution.

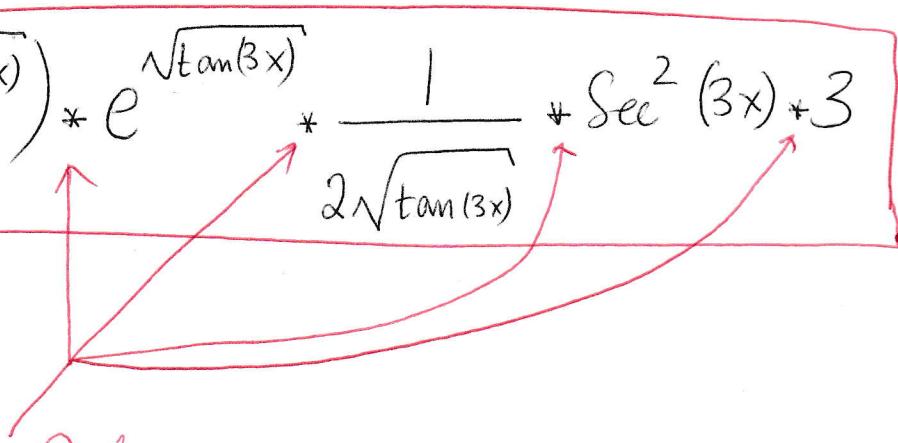
let $u = \sqrt{x}$, then $du = \frac{1}{2\sqrt{x}} dx$, so $\frac{dx}{\sqrt{x}} = 2du$.

So,

$$\begin{aligned} \int \frac{dx}{\sqrt{x}(1+x)} &= \int \frac{2du}{1+u^2} = 2 \int \frac{1}{1+u^2} du \\ &= 2 \arctan(u) + C \\ &= \boxed{2 \arctan(\sqrt{x}) + C} \end{aligned}$$

2) Differentiate the function:

$$y = \cosh(e^{\sqrt{\tan(3x)}})$$

$$\frac{dy}{dx} = \boxed{\text{Sinh}\left(e^{\sqrt{\tan(3x)}}\right) * e^{\sqrt{\tan(3x)}} * \frac{1}{2\sqrt{\tan(3x)}} * \text{Sec}^2(3x) * 3}$$


4 Chain Rule.

3) Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{x}{\sin^{-1}(4x)}$$

$$\lim_{x \rightarrow 0} \frac{x}{\sin^{-1}(4x)} \quad (\text{form } \frac{0}{0}) \stackrel{\text{(L.H)}}{=} \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(x)}{\frac{d}{dx}(\sin^{-1}(4x))}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\frac{4}{\sqrt{1-(4x)^2}}} = \frac{1}{\frac{4}{\sqrt{1}}} = \boxed{\frac{1}{4}}$$