Solution

Math 133, Quiz #4

(Some formulas: $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$, $(\tan^{-1} x)' = \frac{1}{1+x^2}$, $(\sec^{-1} x)' = \frac{1}{x\sqrt{x^2-1}}$)

1. Find y' if

$$y(x) = (2 \sin x)^x$$

$$\ln(y(x)) = x \cdot \ln(2\sin x) \xrightarrow{d/dx} \frac{y'(x)}{y(x)} = 1 \cdot \ln(2\sin x) + x \cdot \frac{2\cos x}{2\sin x}$$

$$\xrightarrow{*y(x)} \boxed{y'(x) = (2\sin x)^x \cdot \left[\ln(2\sin x) + x\frac{\cos x}{\sin x}\right]}$$

Logarithmic differentiation

2. Evaluate the integrals

(a)

$$\int_2^3 \frac{3}{t \ln t}\, dt$$

Let $u = \ln t$, $du = \frac{1}{t} dt$.

$$\int_2^3 \frac{3}{t \cdot \ln t} dt = \int_{\ln 2}^{\ln 3} \frac{3}{u} \cdot du = 3 \ln|u| \Big|_{\ln 2}^{\ln 3}$$

$$= \boxed{3 \ln|\ln(3)| - 3 \ln|\ln 2|}$$

(b)

$$\int \frac{t}{1+t^4}\, dt.$$

Let $u = t^2$, $du = 2t\, dt$

$$\int \frac{t}{1+(t^2)^2} dt = \int \frac{\frac{1}{2} du}{1+u^2} = \frac{1}{2} \int \frac{1}{1+u^2} du$$

$$= \frac{1}{2} \arctan(u) + C = \boxed{\frac{1}{2} \arctan(t^2) + C}$$