

Solution

Math 133, Sections 6-10

Quiz 3

September 18, 2018

- 1) Use logarithmic differentiation to find the derivative of

$$y = \sqrt{\frac{x-1}{x^4+1}}$$

Let $y(x) = \sqrt{\frac{x-1}{x^4+1}} = (x-1)^{\frac{1}{2}}(x^4+1)^{-\frac{1}{2}}$. Then

$$\begin{aligned}\ln(y(x)) &= \ln((x-1)^{\frac{1}{2}}) + \ln((x^4+1)^{-\frac{1}{2}}) \\ &= \frac{1}{2} \ln(x-1) - \frac{1}{2} \ln(x^4+1),\end{aligned}$$

$$\frac{d}{dx} \rightarrow \frac{y'(x)}{y(x)} = \frac{1}{2} \cdot \frac{1}{x-1} - \frac{1}{2} \cdot \frac{1}{x^4+1} \cdot 4x^3$$

$$\xrightarrow{*y(x)} \boxed{y'(x) = \sqrt{\frac{x-1}{x^4+1}} \left(\frac{1}{2} \cdot \frac{1}{x-1} - \frac{2x^3}{x^4+1} \right)}$$

2) When a cold drink is taken from a refrigerator, its temperature is 5°C . After 25 minutes in a 20°C room its temperature has increased to 10°C . Set up the formulas, but do not solve:

(a) What is the temperature of the drink after 50 minutes?

(b) When will the temperature be 15°C ?

By Newton's law of Cooling: $\frac{dT}{dt} = K(T-T_s) = K(T-20)$

Thus, $\int \frac{dT}{T-20} = \int k dt$, and so $\ln |T-20| = kt + C$.

Now, $t=0 \Rightarrow T=5^{\circ}\text{C}$, so $C=\ln |-15|=\ln(15)$, and

$t=25 \Rightarrow T=10^{\circ}\text{C}$, so $\ln |10-20|=K(25)+\ln(15)$, so

$$25K = \ln(10)-\ln(15) = \ln\left(\frac{10}{15}\right) = \ln\left(\frac{2}{3}\right) \Rightarrow K = \frac{1}{25} \ln\left(\frac{2}{3}\right)$$

Thus, $\ln |T-20| = \frac{1}{25} \ln\left(\frac{2}{3}\right) \cdot t + \ln(15)$, and

$$|T-20| = e^{\frac{1}{25} \ln\left(\frac{2}{3}\right)t} * 15.$$

Note Since room temp = 20°C , $T-20 \leq 0$, So $|T-20| = -T+20$.

$$\text{Therefore, } -T+20 = 15e^{\frac{1}{25} \ln\left(\frac{2}{3}\right)t} \Rightarrow T(t) = 20 - 15e^{\frac{1}{25} \ln\left(\frac{2}{3}\right)t}$$

(a) $T(50) = 20 - 15e^{\frac{1}{25} \ln\left(\frac{2}{3}\right) \cdot 50} \quad ^{\circ}\text{C}$

(b) $15 = 20 - 15e^{\frac{1}{25} \ln\left(\frac{2}{3}\right)t} \Rightarrow \frac{15-20}{-15} = e^{\frac{1}{25} \ln\left(\frac{2}{3}\right)t} \Rightarrow \frac{1}{25} \ln\left(\frac{2}{3}\right)t = \ln\left(\frac{1}{3}\right)$

So $t = \frac{\ln\left(\frac{1}{3}\right)}{\frac{1}{25} \ln\left(\frac{2}{3}\right)}$