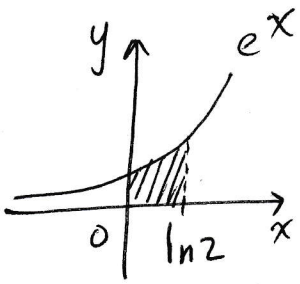
Name: Solution **Section:** 11

Clear your desk of everything except pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. (3 points) What is the area of the region bounded by the graph of $y = e^x$ and the x-axis between $x = 0$ and $x = \ln 2$?



$$A = \int_0^{\ln(2)} e^x dx = e^x \Big|_0^{\ln 2}$$
$$= e^{\ln(2)} - e^0$$
$$= 2 - 1$$
$$= \boxed{1}$$

2. (3 points) Evaluate the following integral.

$$\int \left(\frac{1}{2}\right)^{\cos(x)} \sin(x)dx$$

Let $u = \cos(x)$,
$-du = \sin(x)\,dx$. Then

$$\int \left(\frac{1}{2}\right)^{\cos x} \sin x\,dx = -\int \left(\frac{1}{2}\right)^u du = -\left(\frac{1}{2}\right)^u \cdot \frac{1}{\ln\left(\frac{1}{2}\right)} + C$$
$$= \boxed{-\frac{\left(\frac{1}{2}\right)^{\cos x}}{\ln\left(\frac{1}{2}\right)} + C}$$

3. (4 points) Find the function $y = y(x)$ which satisfies the initial value problem

$$\frac{dy}{dx} = \frac{\sin x}{2y} \qquad y(0) = -2$$

$$2y\,dy = \sin x\,dx$$
$$\Rightarrow \int 2y\,dy = \int \sin x\,dx$$
$$\Rightarrow y^2 = -\cos(x) + C$$
$$(-2)^2 = -\cos(0) + C$$
$$\Rightarrow C = 4 + 1 = 5$$

So, $y^2 = 5 - \cos(x)$, and

$$y = \pm\sqrt{5 - \cos(x)}$$

Since $y(0) = -2 < 0$, we have

$$\boxed{y(x) = -\sqrt{5 - \cos x}}$$