Math 133, Quiz #3

1. Find the inverse function of

$$f(x) = \frac{1+5x}{2-3x}$$

Let $y = \frac{1+5x}{2-3x}$. Then $y(2-3x) = 1+5x$

$\Rightarrow 2y - 3xy - 5x - 1 = 0$

$\Rightarrow x(-3y-5) = 1-2y$

$\Rightarrow x = \frac{1-2y}{-3y-5} = \frac{2y-1}{3y+5}$.

Thus, $\boxed{f^{-1}(x) = \frac{2x-1}{3x+5}}$

2. If $g(x) = x + \sqrt{x}$, what is $g^{-1}(2)$?

Note that $g(1) = 1 + \sqrt{1} = 1+1 = 2$. Thus, (if we assume g is one-to-one) we have

$$\boxed{g^{-1}(2) = 1}$$

3. Find

$$\lim_{x \to +\infty} e^{5-x^5}$$

As $x \to +\infty$, $5 - x^5 \to -\infty$. Therefore

$$\lim_{x \to \infty} e^{5-x^5} = e^{-\infty} = \boxed{0}$$

4. Let $h(x) = \sqrt{\log_{10} x}$. Calculate $h'(x)$ and write your answer using only the natural logarithm, not logarithm to the base 10.

Note that $h(x) = \left(\log_{10}(x)\right)^{\frac{1}{2}}$

$$= \left(\ln(x)\right)^{\frac{1}{2}} \cdot \left(\ln(10)\right)^{-\frac{1}{2}}.$$

Thus, $h'(x) = \frac{1}{2}\left(\ln(x)\right)^{-\frac{1}{2}} \cdot \frac{1}{x} \cdot \left(\ln(10)\right)^{-\frac{1}{2}}$

$$= \boxed{\frac{1}{2x\sqrt{\ln(x) \cdot \ln(10)}}}$$