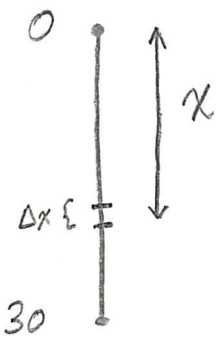
Math 133, Sections 6-10 Solution Quiz 2 September 12, 2018

1) A 1600-lb elevator is suspended by a 200-ft cable that weighs 10 lb/ft. How much work is required to raise the elevator from the basement to the third floor, a distance of 30 ft?.

The work required to lift the elevator & bottom 170 ft of cable is $W_1 = \text{Weight}_{\text{Elevator}} * \text{distance} + W_{\text{bottom cable}} * \text{distance}$

$= 1600 * 30 + 10(170) * 30 = 99{,}000 \text{ ft}\cdot\text{lb}$

<u>For the top 30 ft of cable</u>: a cross-section of length $\Delta x$ ft, $x$ ft away from the 3rd floor,




weighs $(\Delta x)\text{ft} * 10 \text{ lbs}/\text{ft} = (10\,\Delta x)$ lbs.

The work needed to lift it is $W(x) = (10\,\Delta x)(x)\text{ft}\cdot\text{lb}$

Thus the total work to lift the cable is then

$$W_{\text{cable}} = \int_0^{30} 10x\,dx = \frac{10x^2}{2}\Big|_0^{30} = 5x^2\Big|_0^{30} = 5(30)^2$$

$$= 5 * 900 = 4{,}500 \text{ ft}\cdot\text{lb}$$

Therefore, the Total work is

$$W = 99{,}000 + 4{,}500 = 103{,}500 \text{ ft}\cdot\text{lb}$$

2) Let $f(x) = x^5 + x^3 + x$. Assume that this function is one-to-one. Determine $(f^{-1})'(-3)$.

Let's first find $(f^{-1})(3)$. To do so, set

$$x^5 + x^3 + x = -3,$$ and note that

a solution to this equation is $\boxed{x = -1}$. Since $f(x)$ is one-to-one, $x = -1$ is the only solution.

Now, $f'(x) = 5x^4 + 3x^2 + 1$. Thus,

$$(f^{-1})'(-3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(-1)} = \frac{1}{5(-1)^4 + 3(-1)^2 + 1} = \boxed{\frac{1}{9}}$$