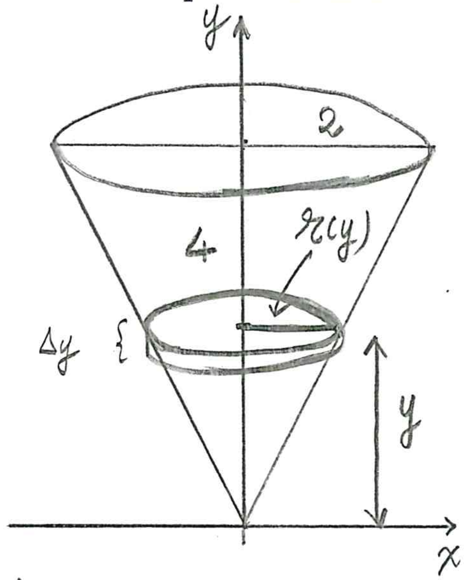
Name: Solution **Section:** 11

Clear your desk of everything except pens, pencils and erasers. **Show all your work.**
If you have a question raise your hand and I will come to you.

1. (5 points) A tank filled with oil is in the shape of a downward pointing cone with its vertical axis perpendicular to ground level. The height of the tank is 4 feet, the circular top of the tank has radius 2 feet, and the oil inside of the tank weighs 50 pounds per cubic foot. How much Work, W, would it take to pump oil from the tank to a level 2 feet above the top of the tank if the tank were completely full? **Set up but do not solve the integral.**



at height y, The cross-section is circular, of radius $r(y)$, with $\frac{r(y)}{y} = \frac{2}{4} \Rightarrow r(y) = \frac{1}{2}y$ ft

Thus its area is $A(y) = \pi r^2(y) = \frac{\pi y^2}{4}$ ft^2, and its volume is $V(y) = \frac{\pi y^2}{4} \Delta y$ ft^3; its weight is $f(y) = \frac{\pi y^2}{4} \Delta y \cdot 50$ lbs; its distance to the pump is $d(y) = (4+2-y)$ ft $= (6-y)$ ft.

Therefore, total work is $\boxed{W = \int_0^4 \frac{\pi y^2}{4} \cdot 50 \cdot (6-y)\, dy \text{ ft-lb.}}$

2. (5 points) Let $f(x) = x^3 + 2x^2 + 3$, for $x > 0$. Find $(f^{-1})'(6)$ at the point $x = 6 = f(1)$.

$f(1) = 6 \Rightarrow f^{-1}(6) = 1.$

$f'(x) = 3x^2 + 4x.$

$$(f^{-1})'(6) = \frac{1}{f'(f^{-1}(6))} = \frac{1}{f'(1)} = \frac{1}{3(1)^2 + 4(1)} = \boxed{\frac{1}{7}}$$