Solution

1) Determine the volume of the solid obtained by rotating the region bounded by $y = \frac{1}{2}x$, $x = 0$ and $y = 1$ about the y-axis.
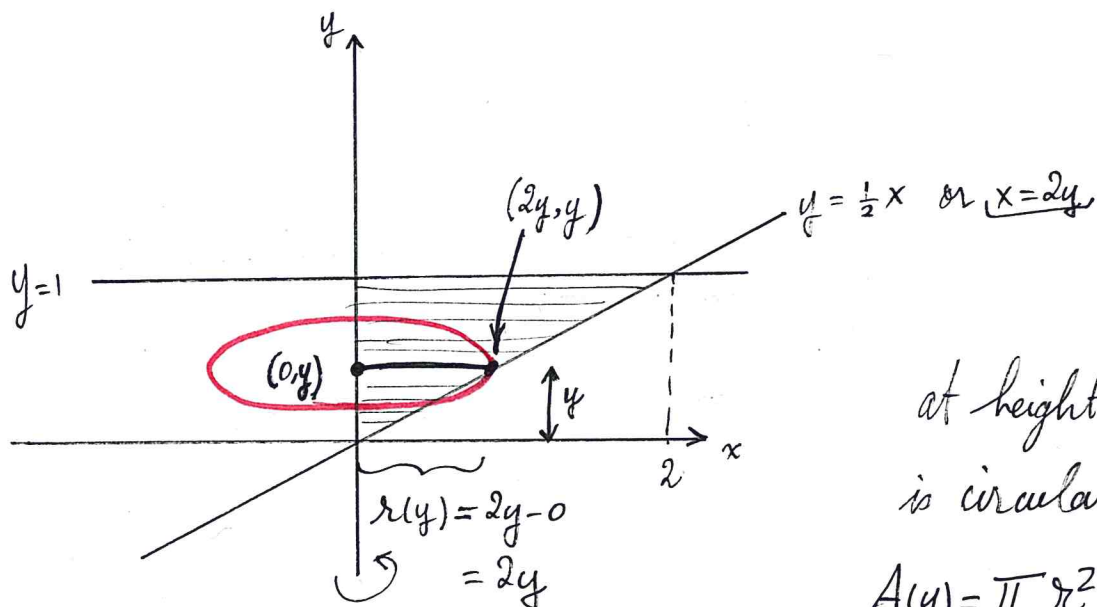
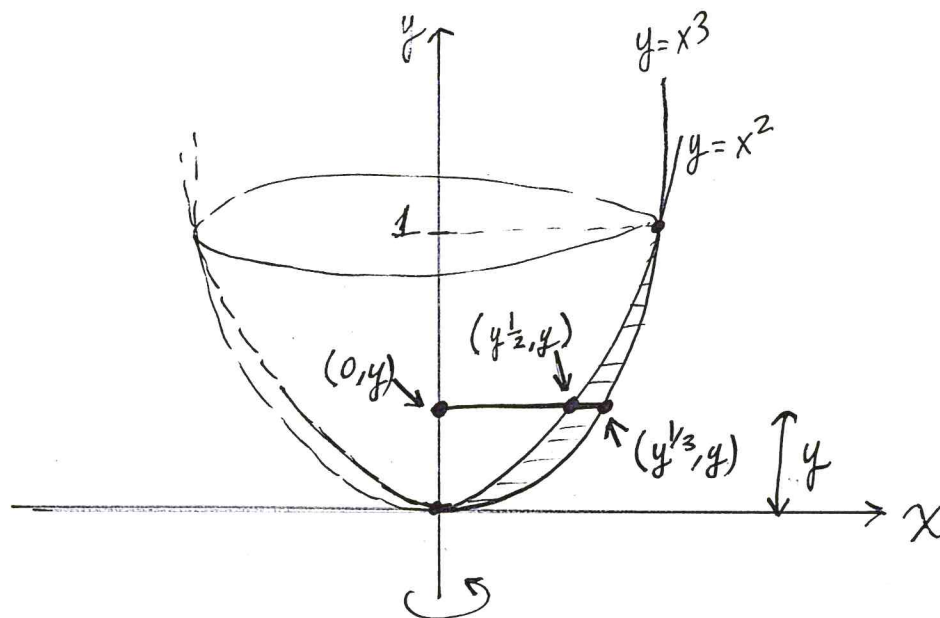at height y, cross-section is circular, of Area

$$A(y) = \pi r^2(y)$$
$$= \pi (2y)^2$$
$$= 4\pi y^2$$

$y = \frac{1}{2}x$ or $x = 2y$

$(2y, y)$

$(0, y)$

$r(y) = 2y - 0 = 2y$

So, $V = \displaystyle\int_0^1 A(y)\, dy = \int_0^1 4\pi y^2\, dy = \left. \frac{4\pi y^3}{3} \right|_0^1 = \frac{4\pi}{3}.$

2) Set up the integral, but do not solve: Determine the volume of the solid bounded by rotating the region bounded by $y = x^2$ and $y = x^3$ about the y-axis.

$$r(y) = y^{\frac{1}{2}} - 0 = y^{\frac{1}{2}}$$
$$R(y) = y^{\frac{1}{3}} - 0 = y^{\frac{1}{3}}$$

at height y, the cross-section is a washer, of Area

$$A(y) = \pi\left(R^2(y) - r^2(y)\right) = \pi\left(y^{2/3} - y\right)$$

So, $V = \displaystyle\int_0^1 A(y)\, dy = \int_0^1 \pi\left(y^{2/3} - y\right) dy$ $\longleftarrow$ $\boxed{\text{Stop Here}}$

Not required $\left\{\begin{array}{l} = \pi\left(\dfrac{3}{5} y^{5/3} - \dfrac{1}{2} y^2\right)\Big|_0^1 \\[2mm] = \pi\left(\dfrac{3}{5} - \dfrac{1}{2}\right) \\[2mm] = \boxed{\dfrac{\pi}{10}}\, _2 \end{array}\right.$