1. Let $G = \text{Sym}(3)$ and $\mathbb{K}$ an algebraically closed field.
   (a) Determine all the simple $\mathbb{K}[G]$-modules (up to isomorphism).
   (b) Compute $J(\mathbb{K}[G])$.
   (c) Let $R = \mathbb{K}[G]/J(\mathbb{K}[G])$. Explicitly find minimal ideals $R_1, R_2, \ldots R_k$ in $R$ with
   $$ R = R_1 \oplus R_2 \oplus \ldots \oplus R_k. $$
   (As before the answers to (1:a) to (1:c) depend on char $\mathbb{K}$.)

2. Let $\mathbb{D}$ be a division ring and $V$ a $\mathbb{D}$-space. Put
   $$ I := \text{FEnd}_\mathbb{D}(V) := \{ f \in \text{End}_\mathbb{D}(V) \mid \dim_\mathbb{D} \text{Im } f < \infty \} $$
   and $R = \mathbb{Z} \text{id}_V + I$. Show that
   (a) $I$ is an ideal in $\text{End}_\mathbb{D}(V)$.
   (b) $R$ is dense on $V$ with respect to $\mathbb{D}$.

3. Let $\mathbb{K}$ be a field and $(V_i, i \in I)$ a family of non-zero finite dimensional $\mathbb{K}$-spaces. Let $J \subseteq I$. Put $V_J = \bigotimes_{j \in J} V_j$. For disjoint subsets $J, K$ of $I$ we can canonically identify $V_J \otimes V_K$ with $V_{J \cup K}$. Let $\overline{J} = I \setminus J$. For each $i \in I$ fix $0 \neq w_i \in V_i$ and put $w_J = \bigotimes_{j \in J} w_j$. Put $W_J = V_J \otimes w_J \leq V_I$. Show that:
   (a) There exists a unique ring monomorphism $\phi_J : \text{End}_\mathbb{K}(V_J) \to \text{End}_\mathbb{K}(V_I)$ with $\phi_J(f)(v \otimes \overline{v}) = f(v) \otimes \overline{v}$ for all $f \in \text{End}_\mathbb{K}(V_J)$, $v \in V_J$ and $\overline{v} \in V_{\overline{J}}$.
   (b) Put $R_J = \text{Im } \phi_J$. Then $W_J$ is a simple $R_J$-submodule of $V_I$.
   (c) If $J$ is finite, then $R_J$ is a simple ring.
   (d) If $J \subseteq K \subseteq I$, then $R_J \subseteq R_K$.
   (e) If $J \subseteq K \subseteq I$, then $W_J \subseteq W_K$.
   (f) Let $R = \bigcup\{R_J \mid J \subseteq I, |J| < \infty\}$. Then $R$ is a simple subring of $\text{End}_\mathbb{K}(V_I)$.
   (g) Let $W = \bigcup\{W_J \mid J \subseteq I, |J| < \infty\}$. Then $W$ is a simple $R$-submodule of $V_I$.
   (h) $R$ is dense on $W$ with respect to $\mathbb{K}$.
   (i) If $|I|$ is infinite, then $R^W \neq \text{End}_\mathbb{K}(W)$ and $R^W \cap \text{FEnd}_\mathbb{K}(W) = 0$.