MTH 309-4 **Linear Algebra I** **F11**

Recitation 11

(A) Let T: $\mathbb{R}^4 \to \mathbb{R}^2$ be a linear function with

$$\ker(T) = \left\{ \left. \begin{pmatrix} 2a \\ b \\ 3a \\ a-b \end{pmatrix} \right| a, b \in \mathbb{R} \right\}.$$

Prove that T is onto.

(B) Let $\mathbf{V}$ and $\mathbf{W}$ be finite dimensional vector spaces. Prove:
(a) There is a one-to-one linear function $T : \mathbf{V} \to \mathbf{W}$ if and only if $\dim(\mathbf{V}) \leq \dim(\mathbf{W})$.
(b) There is an onto linear function $S : \mathbf{V} \to \mathbf{W}$ if and only if $\dim(\mathbf{V}) \geq \dim(\mathbf{W})$.

(C) Consider the matrix

$$A = \begin{bmatrix} a_{11} & 0 & 0 & \dots & 0 \\ a_{21} & a_{22} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix}$$

Prove by induction that $\det(A) = a_{11}a_{21}\dots a_{nn}$.

(D) Let A be an $n \times n$-matrix. Show:
(a) If $A^k = \mathbf{0}$ for some positive integer k, then $\det(A) = 0$.
(b) If $A^k = I_n$ for some positive integer k, then $\det(A) = \pm 1$.