MTH 309-4 **Linear Algebra I** **F11**

Recitation 8

(A) Let A be an $m \times n$ matrix and $L_A : \mathbb{R}^n \to \mathbb{R}^m$ the linear function defined by: $L_A(v) = Av$ for all $v \in \mathbb{R}^n$. Prove that the following conditions are equivalent:

(a) L_A is onto.

(b) The columns of A span $\mathbb{R}^m$.

(c) For all $b \in \mathbb{R}^m$ the linear system $Ax = b$ is solvable.

(B) Let A be an $m \times n$ matrix and $L_A : \mathbb{R}^n \to \mathbb{R}^m$ the linear function defined by: $L_A(v) = Av$ for all $v \in \mathbb{R}^n$. Prove that the following conditions are equivalent:

(a) L_A is 1-1.

(b) The columns of A are linearly independent.

(c) The homogeneous linear system $Ax = \mathbf{0}$ has only the trivial solution.

(C) Given positive integers m and n and an $m \times m$-matrix B. Let $T : \mathbb{M}(m, n) \to \mathbb{M}(m, n)$ be the function defined by $T(A) = BA$ for all $A \in \mathbb{M}(m, n)$.

(a) Prove that T is a linear function.

(b) Find m and n and B such that T is not 1-1.

(D) Given positive integers m and n and an $m \times m$-matrix B. Let $T : \mathbb{M}(m, n) \to \mathbb{M}(m, n)$ be the function defined by $T(A) = BA$ for all $A \in \mathbb{M}(m, n)$. Prove that T is an isomorphism if and only if B is an invertible matrix.