MTH 309-4 **Linear Algebra I** **F11**

Recitation 5

(A) Consider the matrices

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & -5 \\ -4 & 0 \end{bmatrix}.$$

Determine whether the list (A, B, C) is linearly independent in $\mathbb{M}(2.2)$.

(B) Let V be a vector space, $(v_1, \ldots, v_n)$ a linearly independent list in V and $w \in V$. Show that if the list $(v_1 + w, \ldots, v_n + w)$ is linearly dependent then $w \in \operatorname{span}(v_1, \ldots, v_n)$.

(C) Let V be a vector space and $(v_1, \ldots, v_n)$ a list in V. Show that the set

$$U = \{(r_1, \ldots, r_n) \in \mathbb{R}^n \mid r_1 v_1 + \ldots + r_n v_n = \mathbf{0}\}$$

is a subspace of $\mathbb{R}^n$. When is $U = \{(0, \ldots, 0)\}$?

(D) *True or False? Justify your answer:*
Let V be a vector space.

(D1) For all $v \in V$ the list (v) is linearly independent.
True – False?
REASON:

(D2) If $B = (v_1, \ldots, v_n)$ is a list in V with $v_i = 0$ for some $1 \leq i \leq n$ then B is linearly dependent.
True – False?
REASON:

(D3) The list $B = (v_1, \ldots, v_n)$ in V linearly dependent if and only if for all $1 \leq i \leq n$ the vector v_i is a linear combination of the list $(v_1, \ldots, v_{i-1}, v_i, \ldots, v_n)$.
True – False?
REASON:

(D4) A list (x, y, z) in $\mathbb{R}^3$ is linearly dependent if and only if $\operatorname{span}(x, y, z)$ is a plane in $\mathbb{R}^3$.
True – False?
REASON:

(D5) If the list $B = (v_1, \ldots, v_n)$ in V is linearly independent then every nonempty sublist B' of B is linearly independent.
True – False?
REASON:

(D6) If $B = (u_1, \ldots, u_n)$ in a list on V such that for all $1 \leq i < j \leq n$, the list (u_i, u_j) is linearly independent in V, then the list B is linearly independent in V
True – False?
REASON: