MTH 309-4 **Linear Algebra I** **F11**

Recitation 3

(A) Let r be real number with $r \neq 1$. Prove by induction that for any integer $n \geq 1$:

$$1 + r + r^2 + \ldots + r^{n1} = \frac{r^n - 1}{r - 1}$$

(B) Prove by induction that $2n < n!$ for all integers $n \geq 4$.

(C) Let $(1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...) = (s_n)$ be the Fibonacci sequence, that is, $s_{n+2} = s_{n+1} + s_n$, where $s_1 = s_2 = 1$. Prove by induction:
(1) $s_1 + s_2 + \ldots + s_n = s_{n+2} - 1$.
(2) $s_2 + s_4 + \ldots + s_{2n} = s_{2n+1} - 1$
(3) $s_1 + s_3 + \ldots + s_{2n-1} = s_{2n}$
(4) $s_1^2 + s_2^2 + \ldots + s_n^2 = s_n s_{n+1}$
(5) $s_n s_{n+2} = s_n^2 + (-1)^n$.

(D) Consider a homogeneous system of linear equations

$$\begin{array}{ccccccccc} a_{11}x_1 & + & a_{12}x_2 & + & \dots & + & a_{1n}x_n & = & 0 \\ a_{21}x_1 & + & a_{22}x_2 & + & \dots & + & a_{2n}x_n & = & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1}x_1 & + & a_{m2}x_2 & + & \dots & + & a_{mn}x_n & = & 0 \end{array}$$

with solution set $S \subseteq \mathbb{R}^n$. Show that S is a subspace of $\mathbb{R}^n$.

(E) Let V be a vector space and $U, W \subseteq V$ be subspaces of V. Show that the set

$$U + W = \{\mathbf{u} + \mathbf{w} \mid \mathbf{u} \in U \text{ and } \mathbf{w} \in W\}$$

is a subspace of V.