MTH 309-4 **Linear Algebra I** **F11**

Recitation 2

(A) State hypothesis and conclusion in the following theorems from calculus:

(A1) *Theorem:* Suppose that the function f is continuous on the closed interval $[a, b]$. Then $f(x)$ assumes every value between $f(a)$ and $f(b)$.
Hypothesis:

Conclusion:

(A2) *Theorem:* If n is a positive integer and a is a real number and if $a > 0$ for even values of n, then

$$\lim_{x \to a} \sqrt[n]{x} = \sqrt[x]{a}.$$

Hypothesis:

Conclusion:

(A3) *Theorem:* Let C be a piecewise smooth simple closed curve that bounds the region R in the plane. Suppose that the functions $P(x, y)$ and $Q(x, y)$ are continuous and have continuous first-order partial derivatives on R. Then

$$\oint_C Pdx + Qdy = \iint_R \left(\frac{\delta Q}{\delta x} - \frac{\delta P}{\delta y}\right) dA$$

Hypothesis:

Conclusion:

(A4) *Theorem:* If f is differentiable at c and is defined on an open interval con- taining c and if $f(c)$ is either a local maximum value or a local minimum value of f, then $f'(c) = 0$.

Hypothesis:

Conclusion:

(A5) *Theorem:* Suppose that a function g has a continuous derivative on $[a, b]$ and that f is continuous on the set $g([a, b])$. Then

$$\int_a^b f(g(x))g(x)dx = \int_{g(a)}^{g(b)} f(u)du.$$

Hypothesis:

Conclusion:

(A6) *Theorem:* Suppose that the function f is defined on the open interval I and that $f(x) > 0$ for all x in I. Then f has an inverse function g, the function g is differentiable, and

$$g'(x) = \frac{1}{f'(g(x))}$$

for all x in the domain of g.

Hypothesis:

Conclusion:

(B) a) Given statements A, B and C. Use truth-tables to show that the statement "A and (B or C)" is equivalent to the statement "(A and B) or (A and C)"

b) Given sets S, T and U. Show that

$$S \cap (T \cup U) = (S \cap T) \cup (S \cap U).$$

(C) Let W be the union of the first quadrant and third quadrant in the xy-plane, that is, let

$$W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \mid x, y \in \mathbb{R}, xy \geq 0 \right\}$$

(a) If $\mathbf{w} \in W$ and $c \in \mathbb{R}$, is $c\mathbf{w} \in \mathbb{R}$? Why ?
(b) Find specific vectors $\mathbf{u}, \mathbf{w}$ in W such that $\mathbf{u} + \mathbf{w} \notin W$.
(c) Is W a subspace of $\mathbb{R}^2$? Why?

(D1) Are the following statements true or false (circle one):

a) e is a real number and $7 < 10$. [true - false]

b) 119 is a prime number and $\sqrt{3}$ is a rational number. [true - false]

c) 119 is not a prime number or $\sqrt{3}$ is a rational number. [true - false]

d) $f(x) = e^x$ and $g(x) = |x|$ are differentiable at $x = 0$. [true - false]

e) $f(x) = e^x$ or $g(x) = |x|$ is differentiable at $x = 0$. [true - false]

(D2) Are the following statements true or false (circle one):

a) If a and b are integers then $a + b$ is an integer. [true false]

b) If $\sum_{i=1}^{\infty}(-1)^i|a_i|$ converges, then $\sum_{i=1}^{\infty}|a_i|$ converges. [true false]

c) If f is continuous at $x = 0$, then f is differentiable there. [true - false]

d) If $\lim_{i\to\infty} a_i = 0$ then $\sum_{i=1}^{\infty} a_i$ converges. [true false]

e) If $x > 5$ and $y > 5$, then $xy > 15$. [true - false]

f) If $x > 5$ or $y > 5$, then $xy > 15$. [true - false]

g) If squares have (only) three sides, then triangles have four sides. [true false]

(D3) Rephrase the following statements as if - then statements:

a) The differentiability of f is sufficient for f to be continuous.

b) k is an even integer whenever $k + 1$ is odd.

c) For all nonzero real numbers b the square b^2 is positive.

d) $nm \geq 1$ provided that n and m are nonzero positive integers.

e) For every positive real number ϵ there is a positive integer n such that $\frac{1}{n} < \epsilon$.