Homework 11

due on 11/28/11

Section	Exercises
6.8	3,4,5
7.2	1,2b
7.3	10,11,12

A. Fill in all the ? in the proof of the following Theorem:

Theorem I. *Let A be an $m \times n$ matrix and B its reduced row echelon form. Let $x_{f_1}, \dots, x_{f_t}$ be the free variables of B and let s be number of non-zero rows of B . Let $(e_1, \dots, e_n)$ be the standard basis for $\mathbb{R}^n$ and let b^k be row k of B. Then $(b^1, \dots, b^s)$ is a basis for* Row$A$ *and $(b^1, \dots, b^s, e_{f_1}, \dots, e_{f_t})$ is basis for $\mathbb{R}^n$.*

Proof. Put

$$D = (b^1, \dots, b^s, e_{f_1}, \dots, e_{f_t})$$

Note that $(b^1, \dots, b^s)$ is the list of non-zero rows of B. By Theorem ? $(b^1, \dots, b^s)$ is a basis for Row$A$. So we just need to show that D is a basis for $\mathbb{R}^n$. Note that s is the number lead variables and so $n = s + t$. Thus D is a list of length n in the ?-dimensional vector space $\mathbb{R}^n$. So by Theorem ?:

(*) D is basis of $\mathbb{R}^n$ if and only if D is linearly independent.

To show that D is linearly independent, let $r_1, \dots, r_s, u_1, \dots, u_t \in ?$ such that

$$(*) \qquad r_1 b^1 + \dots r_s b^s + u_1 e_{f_1} + \dots u_t e_{f_t} = ?$$

Let $1 \leq k \leq s$ and let b_{kl_k} be the leading 1 in b^k. Then b_{kl_k} is the only non-zero entry in Column l_k of B and so the l_k entry of b^j is ? for all $1 \leq j \leq s$ with $j \neq k$. Since x_{l_k} is a leading variable, $l_k \neq f_j$ for all $1 \leq j \leq t$ and so also the l_k entry of e_{f_j} is ?. Thus the l_k entry of the linear combination on the left side of the equation (*) is ?. Hence ? = 0 for all $1 \leq k \leq s$. Thus (*) implies

$$u_1 e_{f_1} + \dots + u_t e_{f_t} = \mathbf{0}$$

Since $(e_1, \dots, e_n)$ is ? this gives $u_j = ?$ for all $1 \leq j \leq t$. Thus D is ?, and so by (*) D is a basis for $\mathbb{R}^n$. $\square$

B. Let $V = \text{span}\Big((1,0,1,1,1), (3,3,0,3,3), (1,1,0,1,1)\Big)$. Find a basis for V and extend it to a basis of $\mathbb{R}^5$. (*Hint:* Use Theorem I to find both bases simultaneously).