MTH 309-4 **Linear Algebra I** **F11**

Homework 10

due on 11/18/11

Section	Exercises
6.4	1,2
6.5	1,2,3,4,6

A. Let $\mathbf{V}$ and $\mathbf{W}$ be vector spaces, let $B = (v_1, \ldots, v_n)$ be basis for $\mathbf{V}$ and let $E = (u_1, \ldots, u_n)$ be a list in W.

(a) Show that $L_E \circ C_B$ is a linear and $(L_E \circ C_B)(v_j) = u_j$ for all $1 \leq j \leq n$.

(b) Let $T : \mathbf{V} \to \mathbf{W}$ be a linear with $T(v_j) = u_j$ for all $1 \leq j \leq n$. Show that $T = L_E \circ C_B$.

B. Let $\mathbf{V}$ and $\mathbf{W}$ be vector spaces with basis B and D respectively.
Let $n = \dim V$ and $m = \dim W$.
For a linear function T from $\mathbf{V}$ to $\mathbf{W}$ let A_T be the matrix of T with respect to B and D.
Define the function $\alpha : \mathrm{L}(\mathbf{V}, \mathbf{W}) \to \mathbb{M}(m, n)$ by $\alpha(T) = A_T$ for all $T \in \mathrm{L}(\mathbf{V}, \mathbf{W})$.
Define the function $\beta : \mathbb{M}(m, n) \to \mathrm{L}(\mathbf{V}, \mathbf{W})$ by $\beta(A) = L_D \circ L_A \circ C_B$ for all $A \in \mathbb{M}(m, n)$.

(a) Show that α is linear.

(b) Show that β is linear.

(c) Show that β is an inverse of α.

(d) Show that the vector space $\mathbf{L}(\mathbf{V}, \mathbf{W})$ is isomorphic to the vector space $\mathbb{M}(m, n)$.

C. Retain the notation from Exercise B. Suppose $B = (v_1, \ldots, v_n)$ and $D = (w_1, \ldots, w_m)$. For $1 \leq i \leq m$ and $1 \leq j \leq n$ let T_{ij} be unique linear function from $\mathbf{V}$ to $\mathbf{W}$ with

$$T_{ij}(v_k) = \begin{cases} w_i & \text{if } k = j \\ \mathbf{0_W} & \text{if } k \neq j \end{cases}$$

for all $1 \leq k \leq n$. Also let A_{ij} be the $m \times n$-matrix whose (i, j) entry is 1 and all other entries are zero. Show that

$$\alpha(T_{ij}) = A_{ij} \text{ and } T_{ij} = \beta(A_{ij})$$