MTH 309-4 **Linear Algebra I/ Practice Exam 2** **F11 10/26/11**

Show all your work

#1. Use induction to prove that $2 \cdot 3^n \geq 7n - 1$ for all positive integers n.

#2. Which of the following lists are linearly independent in the indicated vector space? (do not justify your answer)

(a) $\big((1,2,3),(1,1,1),(2,3,4)\big)$ in $\mathbb{R}^3$.

(b) $\left(\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}, \begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}, \begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix}\right)$ in $\mathbb{M}(2,2)$.

(c) $(1, \sin^2 x, \cos^2 x)$ in $\mathbf{F}(\mathbb{R})$.

#3. Let $A = \begin{bmatrix}1 & 0 & 1 & 0 & 1\\ 0 & 1 & 2 & 0 & 4\\ 0 & 1 & 2 & 1 & 5\\ 0 & 1 & 2 & 0 & 4\end{bmatrix}$. Find bases for $\mathrm{Col}A, \mathrm{Row}A$ and $\mathrm{Nul}A$.

#4. Let $\mathbf{V}$ be a vector space, I a set and $x, y \in I$. Define the function $T : F(I, \mathbf{V}) \to V$ by $T(f) = f(x) + f(y)$ for all $f \in F(I, \mathbf{V})$. Show that T is linear.

#5. True or false (do not justify your answer)

(a) Every vector space has a basis.

(b) Let $\mathbf{V}$ be a vector space with $\dim \mathbf{V} = 10$. Then any list of length 11 in V spans $\mathbf{V}$.

(c) If $\mathbf{W}$ is a 7-dimensional subspace of a 7-dimensional vector space $\mathbf{V}$, then $V = W$.

(d) If $\mathbf{U}$ and $\mathbf{W}$ are subspace of a vector space $\mathbf{V}$, then $\dim \mathbf{U} + \dim \mathbf{W} \leq \dim \mathbf{V}$.

(e) Let $(v_1, \ldots, v_n)$ be a list in the vector space $\mathbf{V}$. Suppose that $(v_1, \ldots, v_n, v)$ is linearly dependent for all $v \in V$. Then $(v_1, \ldots, v_n)$ spans V.

#6. Let $\mathbf{V}$ be a vector space, $(v_1, \ldots, v_n)$ a list in V and $v \in V$. Prove that the following two statements are equivalent:

(a) $(v_1, \ldots, v_n, v)$ is linearly dependent.

(b) $(v_1, \ldots, v_n)$ is linearly dependent or $v \in \mathrm{span}(v_1, \ldots, v_n)$

#7. Define the function $T : \mathbb{P} \to \mathbb{P}$ by $T(a_0 + a_1 x + \ldots + a_n x^n) = a_1 + a_2 x + \ldots + a_n x^{n-1}$.

(a) Is T linear?

(b) Is T 1-1?

(c) Is T onto?

(Justify all you answers)