

Show all your work

#1. Determine the solution set of

$$\begin{array}{rrrrrr} x_1 & + & 2x_2 & + & 3x_3 & = & 4 \\ 2x_1 & + & 4x_2 & + & 7x_3 & = & 9 \\ -x_1 & + & x_2 & - & x_3 & = & 1 \end{array}$$

#2. Determine the solution set of

$$\begin{array}{rrrrrrrr} x_1 & + & 2x_2 & & & + & 3x_4 & - & 2x_5 & & = & 4 \\ & & & & x_3 & + & x_4 & + & x_5 & & = & 9 \\ & & & & & & & & & x_6 & = & -3 \end{array}$$

#3. For  $a, b$  in  $\mathbb{R}$  define  $a \oplus b = a + b$  and  $a \odot b = ab^2$ . Is  $\mathbb{R}$  a vector space with these operations?

#4. Let  $a, b, c$  be vectors in a vector space  $V$ . Show that

$$2((4a + 7c) + b) = (8a + 2b) + 14c.$$

(Show all your steps. In each step use at most one of the vector space axioms, and indicate which axiom you are using)

#5. Let  $I$  be a set,  $a$  a fixed element of  $I$  and put

$$W = \{f \in F(I) \mid f(a) = 0\}.$$

Show that  $W$  is a subspace of  $F(I)$ .