MTH 309-4 **Linear Algebra I/ Exam 2** **F11 11/04/11**

Show all your work

Your Name: ____________________________

#1. Use induction to prove that

$$1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

for all positive integers n.

#2. Which of the following lists are linearly independent in the indicated vector space? (do not justify your answer)

(a) $\big((1,2,3,4),(1,0,1,0),(1,0,3,1)\big)$ in $\mathbb{R}^4$.

(b) $\left(\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}, \begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}, \begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\right)$ in $\mathbb{M}(2,2)$.

(c) $(x^2+x+1, x^2-1, 3x^2+x-1)$ in $\mathbb{P}_2$.

#3. Let $A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 2 & 2 & 2 & 0 & 0 & 2 \\ 1 & 1 & 1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$. Find bases for $\mathrm{Col}A, \mathrm{Row}A$ and $\mathrm{Nul}A$.

#4. Let $\mathbf{V}$ be a vector space, I a non-empty set and $x, y \in I$. Define the function $T : F(I, \mathbf{V}) \to V$ by $T(f) = f(x) - f(y)$ for all $f \in F(I, \mathbf{V})$.

(a) Show that T is linear.

(b) If $x \neq y$, show that T is onto.

#5. True or false (do not justify your answer)

(a) Every finite dimensional vector space has a basis.

(b) Let $\mathbf{V}$ be a 12-dimensional vector space. Then $\mathbf{V}$ has a unique 6-dimensional subspace.

(c) Let $\mathbf{V}$ be a vector space, $(v_1, \ldots, v_n)$ a linearly independent list in $\mathbf{V}$ and $(w_1, \ldots, w_m)$ a spanning list of $\mathbf{V}$. Then there exists a sublist $(u_1, \ldots, u_l)$ of $(w_1, \ldots, w_m)$ such that $(v_1, \ldots, v_n, u_1, \ldots, u_l)$ is a basis of $\mathbf{V}$.

(d) Let $\mathbf{V}$ be a vector space with $\dim \mathbf{V} = 4$. Then there exists a spanning list of $\mathbf{V}$ which has length 7.

(e) If $(v_1, \ldots, v_n)$ is a linearly dependent list in the vector space $\mathbf{V}$, then v_n is a linear combination of $(v_1, \ldots, v_{n-1})$ in $\mathbf{V}$.