Show all your work

#1. Determine the solution set of

$$
\begin{array}{rcrcrcl}
x_1 & + & 2x_2 & - & 2x_3 & = & 1 \\
x_1 & + & 3x_2 & + & x_3 & = & 6 \\
2x_1 & + & 4x_2 & + & x_3 & = & 3
\end{array}
$$

#2. Determine the solution set of

$$
\begin{array}{rcrcrcrcrcrcr}
x_1 & + & 2x_2 & - & 4x_3 & & & + & 7x_5 & & & = & -2 \\
 & & & & & x_4 & - & & x_5 & & & = & 4 \\
 & & & & & & & & & x_6 & = & 12
\end{array}
$$

#3. For a, b in $\mathbb{R}$ define $a \oplus b = a + b$ and $a \odot b = a^2 b$. Is $\mathbb{R}$ a vector space with these operations?

#4. Let V be a vector space, $r, s \in \mathbb{R}$ and $v, w \in V$. Prove that

$$(r - s)(v - w) = (rv - sv) + (sw - rw).$$

At each step use only one Definition, Theorem or Axiom and indicated exactly which Definition, Theorem or Axiom you are using.

#5. Let V be the vector space and U and W subspaces of V. Prove that $U \cap W$ is a subspace of V.