Review Problems for MTH 234

1. Let $\mathbf{A} = -5\mathbf{i} + \mathbf{k}, \mathbf{B} = 2\mathbf{i} + 10\mathbf{j} + \sqrt{17}\mathbf{k}$. Find $\mathbf{A} \cdot \mathbf{B}$, $\mathbf{A} \times \mathbf{B}$, $|\mathbf{A}|$ and $\text{proj}_{\mathbf{A}}\mathbf{B}$.
2. Find the distance between the point $P(1, 2, 3)$ and the plane $5x - 3y + z = 5$.
3. Find the equation of the line perpendicular to $5x - 3y + z = 5$ and through the point $Q(1, 2, 3)$. Compute the point of intersection between this line and the plane.
4. Find the parametric equations for the line that is tangent to the curve $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j} + \sin(2t)\mathbf{k}$ at $t_0 = \frac{\pi}{2}$. Find the unit tangent vector $\mathbf{T}(t)$.
5. Let $w = w(x, y)$, $x = x(u, v)$, $y = y(u, v)$. Write out the formula for $\frac{\partial w}{\partial u}$. For $w = x^2 + yx^{-1}$, $x = 3u - 5v + 1$ and $y = 5u - v + 3$, find $\frac{\partial w}{\partial u}$ in terms of u and v only.
6. Consider $x^6yz^3 + 4x + 5y + 10z - 20 = 0$. Find $\frac{\partial x}{\partial y}$. Evaluate it at $P_0(1, 1, 1)$.
7. Let $f(x, y) = xy+x^2+5y^2$. Find $L(x, y)$ at $P_0(1, 1)$. Find an estimate of the error $E(x, y)$ on $R : |x-1| \leq \frac{2}{10}, |y-1| \leq \frac{2}{10}$.
8. Find the derivative of $f(x, y, z) = \ln(xy) + yz + zx$ at $P_0(1, -1, 2)$ in the direction of $\mathbf{v} = \sqrt{8}\mathbf{i} + 3\mathbf{j} - \sqrt{8}\mathbf{k}$.
9. Find the equation of the tangent plane to the level surface $\ln(xy) + yz + xz + 1 = \ln(2) + 4$ at the point $P_0(2, 1, 1)$.
10. Find all saddle points, all local maxima and local minima for $f(x, y) = x^3 + 3xy + y^3$.
11. Find the area inside the cardioid $r = 1 + \cos(\theta)$ and outside the circle $r = 1$.
12. Evaluate $\int_0^1 \int_0^1 \int_0^{\sqrt{y}} 2xze^{zy^2}\, dxdydz$. Change the order of integration of $\int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} dzdydx$ into $dydzdx$. Do not evaluate the integral.
13. Set up, do not evaluate, an integral in cylindrical coordinates for the volume of the solid D. The base of D is $z = 0$, the top is in the plane $z = 4 - y$, the sides are given by $r = 2\sin(\theta)$.
14. Use spherical coordinates to set up an integral for the volume of the solid bounded above by $z = 1$ and bounded below by the cone $z = \sqrt{x^2 + y^2}$.
15. Evaluate $\int_C f(x, y, z)ds$ when C is given by $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j} + 6t\mathbf{k}$, $(0 \leq t \leq 2\pi)$ and $f(x, y, z) = x + y + z$.
16. Find the work done by $\mathbf{F} = -3y\mathbf{i} + 3x\mathbf{j} + (x + y)\mathbf{k}$ over the curve $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j} + 4t\mathbf{k}$, $(0 \leq t \leq 2\pi)$ in the direction of increasing t.
17. Consider $\mathbf{F} = (2xy^3z^4 + y)\mathbf{i} + (3x^2y^2z^4 + 2yz + x)\mathbf{j} + (4x^2y^3z^3 + y^2)\mathbf{k}$. Show that $\mathbf{F}$ is conservative. Find f so that $\nabla f = \mathbf{F}$. Evaluate $\int_{(0,0,0)}^{(1,1,1)} \mathbf{F} \cdot d\mathbf{r}$.
18. Quote the circulation form of Green's Theorem. Use it to find $\oint_C (2x + y^2 + 2y)dx + (2xy + 3y + 5x)dy$ where C is the boundary of the triangle bounded by $y = 0$, $x = 0$ and $x + y = 4$.
19. Find the area of the surface cut from $z = 10 + 4xy$ by the cylinder $x^2 + y^2 = 4$.
20. Use the divergence theorem to evaluate $\iint_S \mathbf{F} \cdot \mathbf{n}d\sigma$, where $\mathbf{F} = x^2\mathbf{i} + y^2\mathbf{j} + 3z\mathbf{k}$ and S is the surface of the cube bounded by $x = \pm 1, y = \pm 1, z = \pm 1$.
21. Integrate the function $f(x, y) = x^3/y$ along the plane curve C given by $y = x^2/2$ for $x \in [0, 2]$, from the point $(0, 0)$ to $(2, 2)$.
22. Find the work done by the force $\mathbf{F} = \langle yz, zx, -xy\rangle$ in a moving particle along the curve $\mathbf{r}(t) = \langle t^3, t^2, t\rangle$ for $t \in [0, 2]$.
23. Find the flow of the velocity field $\mathbf{F} = \langle xy, y^2, -yz\rangle$ from the point $(0, 0, 0)$ to the point $(1, 1, 1)$ along the curve of intersection of the cylinder $y = x^2$ with the plane $z = x$.
24. Find the flux of the field $\mathbf{F} = \langle -x, x - y\rangle$ across the loop C given by the circle $\mathbf{r}(t) = \langle a\cos(t), a\sin(t)\rangle$ for $t \in [0, 2\pi]$.

25. (a) Is the field $\mathbf{F} = \langle y\sin(z), x\sin(z), xy\cos(z)\rangle$ conservative?
    (b) If yes, then find the potential function.
    (c) Compute $I = \int_C y\sin(z)dx + x\sin(z)dy + xy\cos(z)dz$, where C is given by $\mathbf{r}(t) = \big\langle \cos(2\pi t), 1+t^5, \cos^2(2\pi t)\pi/2\big\rangle$ for $t \in [0,1]$.

26. Show that the differential form in the integral below is exact,

$$\int_C \left[3x^2dx + \frac{z^2}{y}dy + 2z\ln y dz\right], y > 0$$

27. Compute

$$\int_{(0,0,0)}^{(1,-1,0)} 2x\cos(z)dx + zdy + (y - x^2\sin(z))dz$$

28. Use Green's Theorem in the plane to evaluate the line integral given by $\oint_C (6y+x)dx + (y+2x)dy$ on the circle C defined by $(x-1)^2 + (y-3)^2 = 4$.

29. Use Green's Theorem in the plane to find the flux of $\mathbf{F} = (x - y^2)\mathbf{i} + (x^2 + y)\mathbf{j}$ through the ellipse $9x^2 + 4y^2 = 36$.

30. Set up the integral for the area of the surface cut from the parabolic cylinder $z = 4 - y^2/4$ by the planes $x = 0, x = 1, z = 0$.

31. Integrate the function $g(x,y,z) = x\sqrt{4+y^2}$ over the surface cut from the parabolic cylinder $z = 4 - y^2/4$ by the planes $x = 0, x = 1$ and $z = 0$.

32. Use Stokes' Theorem to find the flux of $\nabla \times \mathbf{F}$ outward through the surface S, where $\mathbf{F} = \langle -y, x, x^2\rangle$ and

$$S = \{x^2 + y^2 = a^2, z \in [0,h]\} \cup \{x^2 + y^2 \le a^2, z = h\}.$$

33. Use the Divergence Theorem to find the outward flux of the field $\mathbf{F} = \langle x^2, -2xy, 3xz\rangle$ across the boundary of the region

$$D = \{x^2 + y^2 + z^2 \le 4, x \ge 0, y \ge 0, z \ge 0\}.$$