Math 234 Final Exam, 5/3/99

1. **(10)** Let $\mathbf{r} = 2t^{3/2}\mathbf{i} + (\cos \pi t)\mathbf{j} + (\sin \pi t)\mathbf{k}$.
   (a) Find the unit tangent $\mathbf{T}$ to $\mathbf{r}(t)$ when $t = 1$.
   (b) Find an integral (DO NOT EVALUATE) that gives the arc length between $\mathbf{r}(0)$ and $\mathbf{r}(1)$.

2. Let $A = (0, 1, 1), B = (1, 2, 1)$ and $C = (4, 3, 2)$.
   (a) **(5)** Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.
   (b) **(5)** Find the cosine of the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$.
   (c) **(5)** Find the area of the triangle with vertices A, B and C.
   (d) **(5)** Find the parametric equation for the line through A and B.
   (e) **(5)** Find the equation of the plane containing A, B and C.

3. **(10)** Let $f(x, y) = \begin{cases} \frac{2x^2-y^2}{x^2+y^2} & (x, y) \neq (0, 0) \\ 2 & (x, y) = (0, 0) \end{cases}$. Show that $f(x, y)$ is not continuous at $(0, 0)$.

4. **(10)** Find the equation of the tangent plane to the surface $x^2 + y^2 - z = -1$ at the point $(1, 1, 3)$.

5. Let $f(x, y) = x^2 - xy + y^2$.
   (a) **(10)** If $g(t) = f(t^3 - 2t, t^2 + t)$, use the Chain Rule to find $g(1)$.
   (b) **(10)** Find the directional derivative of $f(x, y)$ at $(-1, 2)$ in the direction $\mathbf{i} - 2\mathbf{j}$.

6. Let $f(x, y) = x^3 + 3xy + 5$.
   (a) **(8)** Find and classify the critical point of f.
   (b) **(12)** Find the global minimum and maximum of f on the triangle with vertices $(0, -1), (0, 3)$ and $(2, -1)$.

7. Given $\int_0^2 \int_{1+y^2}^5 ye^{(x-1)^2} dx\, dy$.
   (a) **(10)** Sketch the region of integration, labelling all curves.
   (b) **(10)** Evaluate the integral by reversing the order of integration.

8. **(15)** Express in spherical coordinates (DO NOT EVALUATE) the integral $\int_{-2}^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^2 z\, dz\, dx\, dy$.

9. **(15)** Find the work done by the force $\mathbf{F} = 2y\mathbf{i} + 2(x + z)\mathbf{j} + 2(y + z)\mathbf{k}$ over the path $\mathbf{r}(t) = t^2\mathbf{i} + (t - 1)\mathbf{j} + t\mathbf{k}$ from $(0, -1, 0)$ to $(1, 0, 1)$

10. **(15)** Use Green's theorem to evaluate the line integral $\oint_C (xy^2 + 3y - 2)dx + (y^2 \cos y + x^2 y)dy$, where C consists of the rectangle bounded by $x = -2$, $x = 2$, $y = 0$ and $y = 1$.

11. **(20)** Find the surface area of the portion of the paraboloid $z = 4 - x^2 - y^2$ that lies above the plane $z = 0$. Hint: evaluate the integral using polar coordinates.

12. **(15)** Let S be the boundary of the semi-sphere $x^2 + y^2 + z^2 \leq 4, z \geq 0$. Let $\mathbf{F} = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$. Use the divergence theorem to evaluate $\iint_S \mathbf{F} \cdot \mathbf{n} d\sigma$.