MTH 234-67 **Multivariable Calculus** **11/07/11**

Review for Exam 3
Justify all your answers

1. Find the absolute minimum of the function $x^4 + y^4 + 4xy$ on the region $0 \le x \le 2$, $-2 \le y \le 0$.

2. Consider the double integral $\int_0^{\ln 3} \left(\int_{e^x}^{3} dy \right) dx$. Sketch the region of integration and write an equivalent double integral with the order of integration reversed. Do not evaluate any of the integrals.

3. Let $f(x)$ be a continuous function on interval $[a, b]$ and $g(y)$ a continuous function on the interval $[c, d]$. Let h be the function defined by $h(x, y) = f(x)g(y)$. Let K be the average value of f on $[a, b]$, L the average value of g on $[c, d]$ and M the average value of h on the rectangle $a \le x \le b, c \le y \le d$. Show that $M = KL$.

4. Use polar coordinates to evaluate $\iint_R \cos(x^2 + y^2)dA$ where R is the semicircular region bounded by the y-axis and the curve $x = \sqrt{\pi^2 - y^2}$.

5. Set up a triple integral (in cartesian coordinates) which gives the volume of the region bounded below by the surface $z = x^2 + 5y^2$ and above by $z = 16 - x^2 - y^2$. Do not evaluate the integral.

6. Evaluate $\int_{x=0}^{x=1} \int_{y=-1}^{y=1} \int_{z=0}^{z=1} (x + 2y + z)dzdydx$.

7. Set up a triple integral (in cylindrical coordinates) for evaluating the volume of the region D that lies inside the cylinder $r = 2$ and outside the cylinder $r = 1$ and whose top lies in the plane $z = 5 - y$ and whose bottom lies in the plane $z = 0$. Do not evaluate the integral.

8. Use spherical coordinates to compute $\iiint_D z^4 dV$ where D is the region

$$x^2 + y^2 + z^2 \le 4, \quad x^2 + y^2 \le 3z^2 \quad \text{and } z \ge 0.$$