Review Problems for the MTH132 Final

#1. Compute the following limits.

(a) $\lim_{x\to -2} \dfrac{x^2-4}{x+2}$

(b) $\lim_{x\to 0} x\cot x$

(c) $\lim_{x\to 1^-} \dfrac{x}{|x-1|}$

(d) $\lim_{x\to 1^+} \dfrac{\sqrt{x^2-2x+1}}{x-1}$

(e) $\lim_{x\to\infty} \dfrac{x^2+2x+7}{2x^2-2x+1}$.

#2. Find the equation of the line normal to the graph of $f(x) = \sec x$ at the point whose x coordinate is $\frac{-\pi}{3}$.

#3. Find the equations of a lines (there are two of them) passing through $(-2,3)$ and tangent to the parabola $y = x^2$.

#4. Use the Intermediate Value Theorem to show that $\cos x = x^2$ has a solution.

#5. Calculate the derivative of $f(x) = \sqrt{3x+1}$ directly from the definition of derivative.

#6. Let $f(x) = x^{\frac{1}{3}} + \frac{1}{x-2}$.

(a) Find the domain of f.

(b) For which values of x is f continuous at x.

(c) For which values of x is f differentiable at x.

#7. Let $f(x) = \frac{x^2}{(x-1)^2}$.

(a) Find the horizontal and vertical asymptotes of the graph of f.

(b) Find the critical points and the intervals where f is decreasing and increasing.

(c) Find the intervals where f is concave upward and the intervals where f is concave downward.

(d) Sketch the graph using the information obtained in a)-c).

#8. State the Mean Value Theorem and use it to show that if $f(0) = 1$ and $f'(x) > 0$ for all x, then $f(x) > 1$ for all $x > 0$.

#9.

(a) Find $\frac{d^2y}{dx^2}$ if $y = x^{\frac{1}{2}} - \sqrt{x^2+1}$.

(b) Find $f'(x)$ if $f(x) = \frac{x^2+1}{\tan x}$.

(c) Find $\frac{dy}{dx}$ and an equation for the tangent line to the graph of $x^3y + xy^3 = 2$ at $(1,1)$.

(d) Find $\frac{d^2}{dx^2}\left(\int_1^{x^2} \sec t\, dt\right)$.

#10. The velocity of a particle moving along the x-axis is given by $v(t) = t^2 - 3t + 2$.

(a) Find the acceleration.

(b) Its position is 4 for $t = 0$. Find the position at the time t.

(c) Find the maximum position for t in $[0,2]$.

#11. Solve the following initial value problems.

(a) $y' = x+1, y(0) = 1.$

(b) $y' = \sqrt{x}, y(1) = 3.$

(c) $y' = x\sqrt{x^2+1}, y(0) = -1.$

#12. A page of a book is to contain a rectangle of printed matter with an area of 30 square inches. If the page is to have a 1-inch margin on the sides and a 2-inch margin at the bottom and the top, find the dimensions of the smallest such page.

#13. An extension ladder is leaning against a wall and is collapsing at the rate of 1 foot per sec. The bottom of the ladder is being pushed toward the wall at the rate of 3 feet per sec. How fast and in which direction is the top of the ladder moving up or down the wall when the ladder is 13 feet long and the bottom is 5 feet from the wall?

#14. Make a reasonable first approximation to the root of $x^3 + 12x - 30 = 0$. Use Newton's Method to obtain two successive additional approximation of the root.

#15. Let $f(x) = x^2 - 2x$ on $[2, 3]$.

(a) Using 5 intervals of equal length and right endpoints, construct an approximation of the integral $\int_2^3 (x^2 - 2x)dx$. Draw a picture of the rectangles whose area your sum represents.

(b) Using summation notation write an approximation sum for $\int_2^3 (x^2 - 2x)dx$ using n subintervals of equal length and the right endpoints of the subintervals.

(c) Using summation formulas, simplify the sum in b) and take its limit as $n \to \infty$.

#16. Evaluate the following integrals:

(a) $\int_1^2 (x^{\frac{2}{3}} + x^7)dx.$

(b) $\int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \sec^2(2x)dx.$

(c) $\int \frac{\sin\sqrt{2x+2}}{\sqrt{2x+2}}dx.$

(d) $\int_0^5 \frac{1}{\sqrt{3x+1}}dx.$

(e) $\int \frac{2-x^2}{(x^3-6x+1)^5}dx.$

#17. Use linear approximation to approximate $\sqrt[3]{25}$.

#18. A woman is 1 mile north of a pavement which runs west to east. She wants to reach a point on the pavement 2 miles east of her current location. The area between the woman and the pavement is grass. She can walk 3 m.p.h. on the grass and 5 m.p.h. on the pavement. She will walk diagonally to a point on the pavement east of her current location and then walk along the pavement the rest of the way. What route takes the least time?

#19. Find the average value of $\sin x$ on $[0, \frac{\pi}{2}]$.

#20. Find the area bounded by the curves $y = \cos x$ and $y = \sin x$ for $0 \leq x \leq 2\pi$.