

Quiz 1

Name: _____

In all problems, you may use symmetry where appropriate and calculations where necessary.

1. Let X be a real random variable given by a PDF (for some a)

$$f_X(x) = \begin{cases} a(1 - x^2), & \text{for } -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the proper value of a that makes this a probability density function.

(ii) Find $\mathbb{E}(X)$

Let $Y = X^2$

(iii) Find the PDF of Y

2. Let $A = \{(x, y) \in \mathbb{R}^2 : 0 \leq y \leq x \leq 1\}$, Let the pair (X, Y) be uniformly distributed on A .

(i) Find the joint density $f_{X,Y}$ of X and Y

(ii) Find the marginal distributions of X and Y

(iii) Find the Expectation of X and Y

(iv) Find the variance of X and Y

(v) Find the covariance of X and Y , $cov(X, Y)$, write the covariance matrix.

(vi) Find the conditional probability of X with respect to Y for any $Y = y$, ie find $f_X(x|Y = y)$

(vi) Find $E(X|Y)$