Probability Review Questions

1. Write down the density function and CDF for a Bernoulli random variable.

   a. Find the Expectation and Variance for this random variable.

   Try doing this in the most general scenario, ie where X takes on two values a, b and has probabilities $p, (1-p)$ respectively.

2. Write down the density function and CDF for a Uniform random variable.

   a. Find the Expectation and Variance for this random variable.

   Again do this as generally as you can.

3. Consider X given by density.

$$f_X(x) = \begin{cases} ax & 0 \leq x \leq b \\ 0 & otherwise \end{cases}$$

   For any given b find a.

   a. Find the Expectation and Variance for this random variable.

4. Consider $X = (X_1, X_2)$ given by density.

$$f_{X_1,X_2}(x,y) = \begin{cases} a(x+2y) & x \geq 0; y \geq 0; x+2y \leq 2b \\ 0 & otherwise \end{cases}$$

   For any given b find a.

   a. Find the marginals for random variable X_1, X_2.
   b. Find the expectation and variance of X_1, X_2
   c. Find the covariance of X_1 and X_2.
   d. Let $Z = X_1 - X_2$, Find the marginal of Z.
   e. Find the density $f_{X_1}(x|Z = b/2)$.
   f. Find $E(X_1|X_2)$ and $E(Z|X_1)$.
   g. Let $W = X_1 + 2X_2$ find $E(X_1|W)$.