

Homework 4

Several bets Suppose UM is playing MSU.

Let U be the score of UM and M be the score of MSU.

Let ω_1 be the event $U - M > 10$.

Let ω_2 be the event $M - U > 10$.

Let ω_3 be the event $U < M \leq U + 10$.

Let ω_4 be the event $M \leq U \leq M + 10$.

Consider the system of 3 bets, each bet costs \$2, and have the following payoffs:

<i>Bet #</i>	ω_1	ω_2	ω_3	ω_4
<i>Bet 1</i>	3	7	0	1
<i>Bet 2</i>	2	0	4	2
<i>Bet 3</i>	2	2	1	3

That is if you buy 1 of bet 1 and ω_1 occurs you have paid \$ 2 and been paid \$ 3 in return.

- 1 Does there exist an arbitrage opportunity? If there is, find a portfolio that makes an arbitrage profit.

Suppose we are considering the same 4 events but the payoffs of the 3 bets are

<i>Bet #</i>	ω_1	ω_2	ω_3	ω_4
<i>Bet 1</i>	4	5	0	1
<i>Bet 2</i>	2	0	4	2
<i>Bet 3</i>	2	2	0	3

- 2 Does there exist an arbitrage opportunity? If there is, find a portfolio that makes an arbitrage profit.

Let $S(t) \equiv$ the price of some security at time t .

- 3 Suppose $S(0) = 70$ and the effective yearly interest rate is $r = .04$. You write a contract to purchase the security in 9 months. What price should you agree to purchase the security at?
- 4 Suppose at 3 months the price of the security has jumped to \$ 80. What is the value/liability of the forward?
- 5 Suppose you wish to purchase the security in 9 months for \$ 60 what should you pay for the contract today?
- 6 Suppose the security pays 1 dividend of \$ 5 in 1 month what should the forward price be at 9 months?
- 7 Suppose the security pays continuous dividends at a rate of 6% (and no discrete dividends) in this case, what should the forward price be at 15 months?

Suppose \$ 1 USD buys ¥ 6 CNY today. Suppose the effective interest rate of a 1 year bond in USD is 1%, and the effective interest rate of a 1 year bond in CNY is 5%.

- 8 Suppose you wish to obtain ¥1200 in 18 months. Today you enter into a contract to purchase them in 18 months. How many dollars do you agree to pay for them at that time?
- 9 Suppose the exchange rate has changed at the end of 18 months so that now \$ 1 USD buys ¥7 CNY what is the value/liability of the contract?