HOMEWORK 3

(1.) $A_s \equiv$ bond issued at time s for cost \$100
A_s may be cashed at time s+1 for \$120.

t=0. * Ask counterparty to purchase 1 bond A_0 for \$100.

t=½. * Issue/ Short 1.09 bonds $A_{1/2}$.
* Collect \$109
* Buy A_0 from counterparty for \$109.
* Ask counterparty to purchase ~~...~~ $\frac{120}{109}$ $A_{1/2}$ bonds.

t=1. * Cash A_0, collect \$120
* purchase $\frac{120}{109}$ $A_{1/2}$ bonds for 109ea.

t=3/2. * Cash $\frac{120}{109}$ $A_{1/2}$ bonds for \$120 ea.

* Payoff bonds shorted at time t=½: \$(1.09)(120)

$$V(3/2) = \frac{120}{109}\$120 - (1.09)\$120 = \$1.31.$$

$\therefore$ We make ~~\$~~ Arbitrage profit of \$1.31.

(2.) (a) $V^{(1)}_{(0)} = \frac{100}{(1+.1)^{1/4}-1}\left(1-\frac{1}{1.1}\right) + \frac{1}{1.1}5{,}000$

$= 377 + 4545.45 = 4922.45$

$\therefore V(0) \lessapprox F$ slightly below Par (but close to Par w/in 2%)

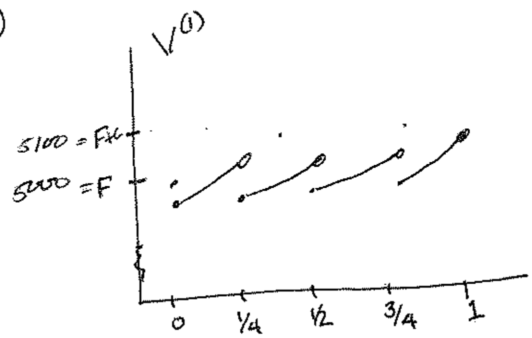
(b) $V^{(2)}_{(0)} = \frac{100}{(1.1)^{1/4}-1}\left(1-\frac{1}{(1.1)^2}\right) + \frac{1}{(1.1)^2}4{,}000$

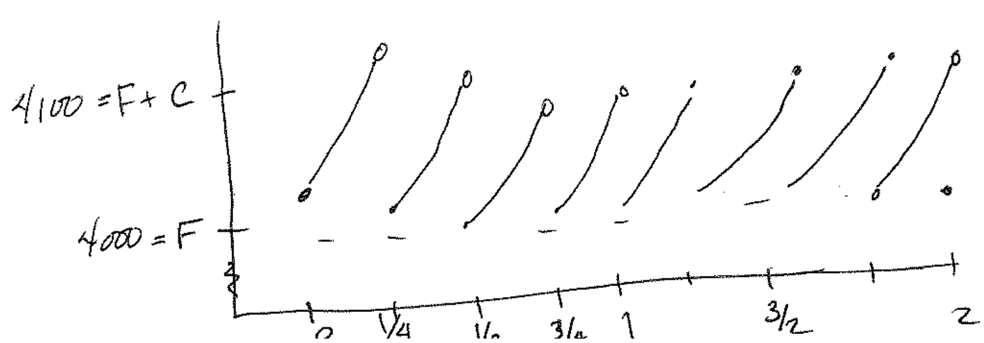
$= 719.73 + 3305.79$

$= 4025.52$

$\therefore V(0) \gtrapprox F$ slightly above Par (close to Par w/in 1%).

(c)





3.) $K_i = \dfrac{S_i(1) - S_i(0)}{S_i(0)} \Rightarrow$ $K_1 = \dfrac{X}{100}$

$K_2 = \dfrac{Y}{100}$.

(a) $1 = \iint_{\mathbb{R}\,\mathbb{R}} f_{XY}(x,y)\,dx\,dy =$

$$= \int_0^{20} \int_0^{10} c\,(x-2y)^2\,dy\,dx =$$

$$= c\,\frac{40{,}000}{3}$$

$$\therefore\ c = \frac{3}{40{,}000}$$

(b) $$f_K(x,y) = f_{XY}(100X, 100Y)\,\left|\det \frac{\partial(X,Y)}{\partial(K_1 \cdot K_2)}\right|$$

$$= \left(f_{XY}(100X, 100Y)\right)\left(100^2\right)$$

$$= \begin{cases} \dfrac{30{,}000}{4}(x-2y)^2 & : 0 < x < .2\ ;\ 0 < y < .1 \\ 0 & : o/w . \end{cases}$$

3.) c)

$$\mu = \begin{pmatrix} \mathbb{E} K_1 \\ \mathbb{E} K_2 \end{pmatrix} = \begin{pmatrix} (\mathbb{E} X) \frac{1}{100} \\ (\mathbb{E} Y) \frac{1}{100} \end{pmatrix}$$

$$\mathbb{E} X = \frac{3}{40{,}000} \left(\frac{400{,}000}{3} \right) = 10$$

$$\mathbb{E} Y = \frac{3}{40{,}000} \left(\frac{200{,}000}{3} \right) = 5$$

$$\mu = \begin{pmatrix} .1 \\ .05 \end{pmatrix}$$

d) $$\mathbb{E} X^2 = \frac{3}{40{,}000} \frac{17\,600\,000}{9} = \frac{440}{3}$$

$$\mathbb{E} Y^2 = \frac{3}{40{,}000} \frac{4\,400{,}000}{9} = \frac{110}{3}$$

$$\mathbb{E} XY = \frac{3}{40{,}000} \frac{4\,000\,000}{9} = \frac{100}{3}$$

$$\text{var } X = 140/3 \Rightarrow \text{var } K_1 = \frac{140}{30000} = .0046$$

$$\sigma_1 = .068$$

$$\text{var } Y = \frac{35}{3} \Rightarrow \text{var } K_2 = \frac{35}{30000}$$

$$\sigma_2 = .034$$

$$\text{cov}(X \cdot Y) = \frac{-50}{3} \Rightarrow \text{cov}(K_1, K_2) = \frac{-50}{30000}$$

$$\rho_{12} = -.721$$

3(d)

covarizance matrix:

$$\Sigma_K = \begin{pmatrix} 140 & -50 \\ -50 & 35 \end{pmatrix} \frac{1}{30,000}$$

(e) $\sigma_2 < \sigma_1 \Rightarrow \rho < \frac{\sigma_2}{\sigma_1}$ is regim of attaining

min risk portfolio w/o short selling.

$\therefore$ min risk portfolio does not require short.

Minimum Varizance Portfolio:

$$W = \frac{\Sigma_K^{-1} \mathbb{1}}{\mathbb{1}^T \Sigma_K^{-1} \mathbb{1}} \quad ; \quad \Sigma^{-1} = \begin{pmatrix} 35 & 50 \\ 50 & 140 \end{pmatrix} 30,000$$

$$= \begin{pmatrix} 85 \\ 190 \end{pmatrix} \frac{1}{275}$$