Homework 2

1 (a) $E(S_n | S_m) = E(X_n + \cdots + X_{m+1} + S_m | S_m)$

$$= E(X_n | S_m) + \cdots + E(X_{m+1} | S_m) + E(S_m | S_m)$$

$$= \frac{7}{2}(m-n) + S_m$$

$$var(S_n | S_m) = \cancel{var(X_n + \cdots + X_{m+1} | S_m)}$$

$$= var(X_n + \cdots + X_{m+1} + \underbrace{S_m}_{const.} | S_m)$$

$$= var(X_n + \cdots + X_{m+1} / S_m)$$

$$= (n-m)\, var(X_n) = (n-m)\frac{35}{12}$$

(b) $\mathbb{E} X_1 = \frac{7}{2}$, $Var X_1 = \frac{35}{12}$; $\sigma = \frac{\sqrt{35}}{2\sqrt{3}} \cong \sqrt{3}$

$$\mathbb{P}(S_{100} > 400) = \mathbb{P}\left(\frac{S_{100} - 350}{10\sqrt{3}} > \frac{400-350}{10\sqrt{3}}\right)$$

$$= \mathbb{P}\left(\frac{S_{100} - 350}{10\sqrt{3}} > 5/\sqrt{3}\right)$$

$$\cong \mathbb{P}(Z > 5/\sqrt{3})$$

$$\cong \mathbb{P}(Z > 2.88) \cong 1 - .998 = .002$$

(c) $\mathbb{P}(S_{100} > 400 \mid S_{50} = 170)$

$$= \mathbb{P}(S_{100} - S_{50} > 230 \mid S_{50} = 170)$$

$$= \mathbb{P}\left(\frac{S_{100} - S_{50} - 175}{\sqrt{150}} > \frac{55}{\sqrt{150}} \;\middle|\; S_{50} = 170\right)$$

$$\approx \mathbb{P}(Z > 4.49) \approx 0$$

(d) $\mathbb{P}(S_{100} > 400 \mid S_{50})$

$$=$$

$$\mathbb{P}\left(\frac{(S_{100} - S_{50}) - 175}{\sqrt{150}} > \frac{(400 - S_{50}) - 175}{\sqrt{150}} \;\middle|\; S_{50}\right)$$

$$\cong$$

$$\mathbb{P}\left(Z > \frac{225 - S_{50}}{\sqrt{150}} \;\middle|\; S_{50}\right)$$

2. (a)

$$V(t) = 700(1+(.05)t)$$

$$730 = 700(1+(.05)t)$$

$$30 = ((.05)t)700$$

$$\frac{600}{700} = \frac{6}{7} = t.$$

(b)

$$V(0) = 650 \; ; \; V(1/2) = 680$$

$$V(t) = 650(1+rt)$$

$$680 = 650(1+r\tfrac{1}{2})$$

$$\frac{68}{65} = 1 + r/2$$

$$r = \frac{6}{65}$$

3.). (a) $V(i/12) = \left(1 + \frac{.05}{12}\right)^{i} 1000$

$V(1) = (1.0512)(1000) = 1051.2$

$V(i/52) = \left(1. + \frac{.04}{52}\right)^{i} 1000$

$= (1.0408)\ (1000) = 1040.8$.

(b)

$1090 = \left(1 + \frac{r}{52}\right)^{52} 1000$

$52\left\{(1.09)^{\frac{1}{52}} - 1\right\} = r$

$r = .08625$

4.) a.) $V = \frac{1{,}000}{.05}\left(1 - \left(\frac{1}{1.05}\right)^{20}\right)$

$= 12{,}462$

(b) Consider a ~~bond~~ Annuity paying \$1000 per year for 20 years valued at \$10000 today what is the effective interest rate?

$$10{,}000 = \frac{1{,}000}{r}\left(1 - \left(\frac{1}{1+r}\right)^{20}\right)$$

$$\Leftrightarrow \quad 10\, r\, (1+r)^{20} = (1+r)^{20} - 1$$

Solve this polynomial to find rate of interest:

Using Wolfram alpha; find a real root:

$r = .0775$

7.75 % effective interest.

The original statement of the problem had the Annuity at time zero valued at \$49000. This is a poorly chosen number since the value of the payments w/o markdown is 20000. This unfortunately leads to negative interest ↯.