Homework 8 – Quiz Name: Key

Let $W_t \equiv$ Standard Brownian Motion

1. Find

$$d\left(W_t \sin W_t\right)$$

$(x \sin x)' = \sin x + x\cos x$

$(x \sin x)'' = 2\cos x - x \sin x$

$$d(W_t \sin W_t) = \frac{1}{2}(2\cos W_t - W_t \sin W_t)dt + (\sin W_t + W_t \cos W_t)\, dW_t$$

2. Find *(Hint: you do not have to calculate the integral to calculate the 'd').*

$$d\left(\int_0^t W_t dW_t\right)$$

$$d\int_0^t W_t\, dW_t = W_t\, dW_t \quad \text{By definition}$$

3. Find *(Hint: you do not have to calculate the integral to calculate the 'd').*

$$d\left(\sin \int_0^t W_t dW_t\right)$$

$$\left(\sin \int_0^t W_t dW_t\right)' = \left(\cos \int_0^t W_t dW_t\right) dW_t\, dW_t$$

$$\left(\sin \int_0^t W_t dW_t\right)'' = \left(-\sin \int_0^t W_t dW_t\right) W_t^2 + \cos \int_0^t W_t dW_t$$

$$d\left(\sin \int_0^t W_t dW_t\right) = \frac{1}{2}\left(-W_t^2 \sin \int_0^t W_t dW_t + \cos \int_0^t W_t dW_t\right)dt + \left(\cos \int_0^t W_t dW_t\right) W_t dW_t$$

4. Let

$$dX_t = \sin(W_t)dt + e^{-t}W_t dW_t \quad dY_t = \sin(W_t)dt + \sigma Y_t dW_t.$$

Find

$$d\left(X_t Y_t\right)$$

$$d(X_t Y_t) = (dX_t)Y_t + X_t\, dY_t + dY_t\, dX_t$$

$$= \left((\sin W_t)Y_t + X_t(\sin W_t) + e^{-t}W_t \sigma Y_t\right)dt$$

$$+ \left(X_t \sigma Y_t + e^{-t}W_t Y_t\right) dW_t$$