

Homework 7

Name: Key.

1. Consider a binomial model $S(0) = 100$ and $r = .01$ and two possible return values $m_1 = .03$ and $m_2 = -.03$.

Find the (time 0) value of an Asian call option with expiry at step 2 with strike $X = 99$. The payoff of the Asian call is:

$$\left(\frac{1}{2}[S(1) + S(2)] - X\right)^+$$

$$100 \begin{matrix} < 103 \\ < 97 \end{matrix} \begin{matrix} \begin{matrix} 106.09 \\ 99.91 \\ 94.09 \end{matrix} \end{matrix} \left| \begin{matrix} \frac{S^{uu} + S^u}{2} = 4.55 \\ \frac{S^{ud} + S^u}{2} = 101.46 \end{matrix} \right| \begin{matrix} \frac{S^{du} + S^d}{2} = 98.46 \\ \frac{S^{dd} + S^d}{2} = 95.55 \end{matrix}$$

$$C_1^u = \frac{1}{1.01} \left\{ \frac{2}{3}(5.55) + \frac{1}{3}(2.46) \right\} = 4.48$$

$$C_1^d = \frac{1}{1.01} \left\{ \frac{2}{3}0 + \frac{1}{3}0 \right\} = 0$$

$$C_0 = \frac{1}{1.01} \left\{ \frac{2}{3}(4.48) + \frac{1}{3}(0) \right\} = 2.96$$

2. Setup a binomial stock price model over 50 time steps over one year with effective interest 1% and log return having risk neutral variance $\sigma^2 = (.02)^2$ at the end of one year. Find the price of a European Call with $S_0 = 40$, strike $X = 40$ and expiry 1 year.

Use the Gaussian approximation of the binomial distribution to approximate the sum.

$$m_u = \frac{.01}{50} + \frac{.02}{\sqrt{50}}; m_d = \frac{.01}{50} - \frac{.02}{\sqrt{50}}; p_u = p_d = \frac{1}{2}; r = \frac{.01}{50}$$

$$k_0 = \left\lceil \frac{\log \frac{S_0}{X(1+m_d)^{50}}}{\log \frac{1+m_u}{1+m_d}} \right\rceil = \left\lceil \frac{\log 1.141}{.0057} \right\rceil = 24 \quad \left\| \begin{array}{l} q_u = p_u \frac{1+m_u}{1+r} \\ = .5014 \\ q_d = .4986 \end{array} \right.$$

$$C_{OE} = 40 \mathbb{P}\left(Z \geq \frac{24 - 25.07}{3.56}\right) - \frac{40}{(1 + \frac{.01}{50})^{50}} \mathbb{P}\left(Z \geq \frac{24 - 25}{(.5) \sqrt{50}}\right)$$

$$= 40 \mathbb{P}(Z \geq -.3) - (39.6) \mathbb{P}(Z \geq -.28)$$

$$= .56.$$