Homework 4

1.) Let $A \in \mathbb{R}^{n \times n}$ be an $n \times n$ real matrix and let $b \in \mathbb{R}^n$ be a n dimensional vector. Let $f(x) = x^T Ax$ map $\mathbb{R}^n$ to $\mathbb{R}$ and let $g(x) = x^T b$.

   a.) Find ∇g

   b.) Find ∇f

   Hint: for part b, write $f = \sum_{ij} x_i A_{ij} x_j$ and take derivative with respect to each variable. Find a matrix formula for the solution. We are not assuming A is symmetric.

Many securities - risk and return

Suppose you are given securities S_1, S_2, S_3 and at time 0, $S_1(0) = 100$, $S_2(0) = 150$ and $S_3(0) = 50$. At time 1 the value of each security takes on a value depending on the outcome of the experiment,

Ω	$S_1(1)$	$S_2(1)$	$S_3(1)$	$\mathbb{P}$
ω_1	120	150	45	1/4
ω_2	110	165	50	1/4
ω_3	100	165	60	1/4
ω_4	90	165	50	1/4

2.) Find the return variables K_i *That is rewrite the table with values of K*

3.) Find the covarance matrix Σ and expected return m of the return variables K_i for $i = 1.2.3$.

4.) For each 2 security submarket $(K_1, K_2), (K_2, K_3), (K_1, K_3)$, find the minimal variance portfolio and the asymptotes of the feasible set. Graph the 3 feasible sets.

5.) For the entire market, find the minimal variance portfolio, and the risk and return of that portfolio. Find the minimal variance line, and asymptotes of minimal variance line.

6.) Compare these subsystems to the entire market system ie graph all systems together. Does the min variance line pass through the set without short selling? Does the min variance portfolio require short selling?

7.) Suppose we add a risk free bond to the above example at return $R = .05$ Find the market portfolio and the capital market line. Does the market portfolio require short selling?