

Homework 3

- 1.) Refer to the notes. For a contract costing \$ 100 at time 0 and paying \$120 at $t = 1$ (at the end of one year). That is $V(0) = 100$ and $V(1) = 120$. We show that if someone values $V(1/2)$ at \$ 110, we may obtain a risk free profit. Show that if someone values $V(1/2)$ at \$ 109 we may also obtain a risk-free profit.
- 2.) Let $V(t)$ be the value of a coupon bond at time t which pays \$ 100 quarterly (4 times per year) at an effective interest of 10% (notice effective interest is per year but payments are quarterly)
 - a) Suppose the bond matures in 1 year at which time it pays a face value (along with the final coupon) of \$ 5000, is it above below or at par?
 - b) Suppose the bond matures in 2 years at which time it pays a face value (along with the final coupon) of \$ 4000, is it above below or at par?
 - c) Plot $V(t)$ for both schemes from a) and b).
- 3.) Let S_1 and S_2 be two stocks, both valued at 100 at time 0, and let $S_1(1) = 100 + X$ and $S_2(1) = 100 + Y$. Where (X, Y) are jointly distributed random variables with density,

$$f_{X,Y}(x, y) = \begin{cases} c(x - 2y)^2 & : 0 \leq x \leq 20; 0 \leq y \leq 10 \\ 0 & : otherwise \end{cases}$$

for c a properly chosen normalization constant.

- a) find c
- b) find the variables K_1, K_2 - that is, find the joint distribution of the return.
- c) Find the return expectation
- d) Find the covariance matrix
- e) Find the minimal variance profile. Does it require short selling?