Homework 2

1.) Consider an experiment of rolling n die in a row - assume the die are standard 6 sided fair die with faces labeled 1 through 6. Let X_i be the face value of the i^{th} flip and for $m < n$ let $S_m = X_1 + \cdots + X_m$.

   a.) Find $\mathbf{E}(S_n|S_m)$ and $\mathbf{E}(S_n^2|S_m)$. Notice from this we can find $var(S_n|S_m)$ (find the value of this variance)

   b.) Use the central limit theorem to estimate $\mathbb{P}(S_{100} > 400)$,

   c.) Use the central limit theorem to estimate $\mathbb{P}(S_{100} > 400|S_{50} = 170)$

   d.) Use the central limit theorem to estimate $\mathbb{P}(S_{100} > 400|S_{50})$

2.) Loans with simple interest

   a.) Suppose the value of the loan with rate 5% at time 0 is 700, at what time does the value equal 730?

   b.) Suppose the value of the loan at time 0 is 650, what is the rate of the loan if its value is 680 at 6 months $t = 1/2$?

3.) Loans with compound interest

   a.) Consider a loan that compounds monthly at a rate of 5% versus a loan that compounds weekly at a rate of 4%. If both accounts are valued at $ 1000 initially which account is worth more at the end of one year?

   b.) Suppose a loan compounds weekly and begins with an initial value of $ 1000 and at the end of a year is worth $1090, what is the rate of interest of the loan?

4.) Annuities

   a.) What is the value of an Annuity which pays 1000 per year for 20 years assuming an interest rate of 5%

   b.) Suppose an Annuity pays 1000 per year for 20 years and is worth 40,000 today, what is the rate of the interest?