Math finance review Exam 2

1.) Forwards – assume a risk free interest rate of 4%

   a.) Suppose today's value of a security is \$70. You write a contract to sell the security at the end of a year, how much do you write the security for?(forward price)

   b.) Suppose, in addition, the security pays dividends of \$1 monthly - starting in 1 month and you sell the security after the final (12th) dividend. How is the forward price modified?

   c.) Suppose after 6 months (after 6th dividend), the value of the stock is \$80. You decide to clear the contract. How much do you pay the holder?

2.) Currency exchange. Suppose USD has an interest rate of 2% and the Mexican Peso has an interest rate of 5%, assume the exchange rate is \$1 USD = \$20 MXN

   a.) Suppose you wish to aquire \$1000 MXN peso in 9 months time. What should you agree to pay for it?

   b.) Suppose at 3 months the exchange rate has moved to \$1 USD = \$19 MXN what is the value of the contract at that time?

3.) Binomial model – Let a stock have initial value $S_0 = 90$. Assume at each month the return of the stock is $m_u = 2\%$ or $m_d = -1\%$. Assume the effective yearly interest rate is %4.

   a. Find the (time 0) value of a European call with expiry time at 6 months with strike price $X = 91$. What is the replicating portfolio at each step?

   b. Find the (time 0) value of a European put with expiry time at 6 months with strike price $X = 91$.

   c. Find the (time 0) value of the Asian Call with expiry 6 months and fixed strike $X = 91$.

   d. Find the (time 0) value of the American Put with expiry in 3 months and strike $X = 89$.

4 Suppose a stock has time 0 value = 90 and implied volatility $\sigma = 5\%$ (per year). If the effective yearly interest rate is 1% find the value of the European Call with strike price $X = 90$ and expiry 9 months using [a] Black Scholes and [b] the Binomial model for $N = 52$ and 365 steps per year. Compare the answers (Use the Gaussian to approximate the value from the binomial model.)

5.) Let W_t be Brownian motion.

   a. Find $d(\sin(t)\cos(W_t))$.

   b. Find
$$d\left(\left[t\int_0^t W_t dt\right]^2\right)$$

   c. Find
$$d\left(\cos(W_t)\left[t\int_0^t W_t dt\right]^2\right)$$