Foreign currency - Forwards -

Consider 2 currencies \$ USD
¥ Yuan

Each currency has a bond rate

$B_\$(t,T) \equiv \left\{ \begin{array}{l} \$\text{ value of security @ time } t \\ \text{ + payoff of } \$1 \text{ at time } T \end{array} \right.$

$B_¥(t,T) \equiv \left\{ \begin{array}{l} ¥\text{ value of security @ time } t \\ \text{ + payoff of } ¥1 \text{ at time } T. \end{array} \right.$

Exchange value

$\rho(t) = \rho(t; ¥, \$) \equiv \$ \text{ value of } ¥1 \text{ @ time } t$ " $= \frac{\$}{¥}$ "

units.

y yuan is worth $\$ \rho_t y$ @ time t

x USD is worth $¥ \frac{x}{\rho_t}$ @ time t.

Consider Forward to buy ¥ 1 at time T,
what do you agree to pay @ time $t=0$

Notice: we can construct a 'forward'.

time $t=0$: (i) Borrow $\$ \rho B_{¥}(0,T)$
in Bond w/ maturity T

(ii) Exchange \$ to ¥ $B_{¥}(0,T)$.

(iii) Buy Bond ~~\$~~ in ¥ w/ maturity T.

time $t=T$: (i) Bond ~~\$~~ in ¥ w/ maturity T matures
collect ¥ 1.

(ii) Pay $\$ \rho_0 \frac{B_{¥}(0,T)}{B_{\$}(0,T)}$

∴ At end of experiment our value is

$$V(T) = ¥ 1 - \$ \rho_0 \frac{B_{¥}(0,T)}{B_{\$}(0,T)}$$

Let us prove $F = \$\rho_0 \dfrac{B_{\yen}(0,T)}{B_{\$}(0,T)}$.

(A) Suppose price $\$P$ is on the market w/ $\$P < \$F$.

$\hat{V}$: * Long forward at price P short V. @ time $t=0$,

* at time $t=T$, (*) pay $\$P$ get $\yen 1$

(*) Short $V \to -V(T)$

$$\therefore \quad \hat{V} = \yen 1 - \$P - \yen 1 + \$\rho_0 \frac{B_{\yen}(0,T)}{B_{\$}(0,T)}$$

$$= \$\left(\rho_0 \frac{B_{\yen}(0,T)}{B_{\$}(0,T)} - P\right)$$

(B) Suppose price P is on the market w/ $\$P > \$F$

$\hat{V}$: * Short forward at price $\$P$ + Long portfolio V, @ $t=0$.

* at time $t=T$, (*) Pay $\yen 1$ collect $\$P$

(*) Long $V \to V(T)$

$$\hat{V} = \$P - \yen 1 + \yen 1 - \$\rho \frac{B_{\yen_0}(0,T)}{B_{\$}(0,T)}$$

$$= \$\left(P - \rho \frac{B_{\yen_0}(0,T)}{B_{\$}(0,T)}\right).$$