3 securities w/ return $K_1\ K_2\ K_3$.

Covariance: $\Sigma = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \frac{1}{100}$;

Return: $m = \begin{pmatrix} 0.0 \\ 0.1 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \frac{1}{10}$.

$$\Sigma^{-1} = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{pmatrix} 25.$$

$$\Sigma^{-1} \mathbb{1} = \begin{pmatrix} 6 \\ 8 \\ 6 \end{pmatrix} 25. \qquad \Sigma^{-1} m = \begin{pmatrix} 4 \\ 8 \\ 8 \end{pmatrix} \frac{25}{10}$$

MVP:

$$W_0 = \frac{\Sigma^{-1}\mathbb{1}}{\mathbb{1}\Sigma^{-1}\mathbb{1}} = \frac{\begin{pmatrix} 6 \\ 8 \\ 6 \end{pmatrix} 25}{(6+8+6)25} = \begin{pmatrix} 6/20 \\ 8/20 \\ 6/20 \end{pmatrix} = \begin{pmatrix} 3/10 \\ 4/10 \\ 3/10 \end{pmatrix}$$

MVL:

$$\begin{pmatrix} \mu \\ 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} m^T \Sigma^{-1} m & m^T \Sigma^{-1} \mathbb{1} \\ m^T \Sigma^{-1} \mathbb{1} & \mathbb{1}^T \Sigma^{-1} \mathbb{1} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \cancel{\frac{24}{4}}\ 6 & 50 \\ 60 & 500 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}$$

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 3 & 25 \\ 25 & 250 \end{pmatrix}^{-1} \begin{pmatrix} \mu \\ 1 \end{pmatrix} = \frac{1}{125} \begin{pmatrix} 250 & -25 \\ -25 & 3 \end{pmatrix} \begin{pmatrix} \mu \\ 1 \end{pmatrix} = \begin{pmatrix} 2\mu - \frac{1}{5} \\ -\frac{1}{5}\mu + \frac{3}{125} \end{pmatrix}$$

∴ Minimum variance line (line 3.20 in book)

$$w = \frac{\lambda_1}{2} \Sigma^{-1} m + \frac{\lambda_2}{2} \Sigma^{-1} \mathbb{1}.$$

$$w = \frac{\lambda_1}{2} \begin{pmatrix} 4 \\ 8 \\ 8 \end{pmatrix} \frac{25}{10} + \frac{\lambda_2}{2} \begin{pmatrix} 6 \\ 8 \\ 6 \end{pmatrix} \frac{25}{}$$

$$= \lambda_1 \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} 5 + \lambda_2 \begin{pmatrix} 30 \\ 40 \\ 30 \end{pmatrix} 5/2$$

$$= \mu \left\{ 10 \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 30 \\ 40 \\ 30 \end{pmatrix} \right\} + \left\{ \begin{pmatrix} 30 \\ 40 \\ 30 \end{pmatrix} \frac{3/2}{25} - \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \cancel{\begin{pmatrix} \\ \\ 30 \end{pmatrix}} \right\}$$

mvl is linear in μ.

$$w = \mu \begin{pmatrix} -5 \\ 0 \\ 5 \end{pmatrix} + \begin{pmatrix} 9/5 - 1 \\ 12/5 - 2 \\ 9/5 - 2 \end{pmatrix}$$

$$= \mu \begin{pmatrix} -5 \\ 0 \\ 5 \end{pmatrix} + \begin{pmatrix} 4/5 \\ 2/5 \\ -1/5 \end{pmatrix}$$

Minimum Variance Portfolio

$$w_0 = \begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix} \frac{1}{10}$$

$$m = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \frac{1}{10} \quad ; \quad \Sigma = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \frac{1}{100}$$

$$\Sigma^{-1} = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{pmatrix} 25$$

(μ, σ) of w_0

$$\mu_0 = w_0^T m = \frac{4 + 2.3}{100} = \frac{1}{10}$$

$$\sigma_0^2 = \frac{1}{\mathbb{1}\, \Sigma^{-1} \mathbb{1}} = \frac{1}{20} \frac{1}{25} = \frac{1}{500} \quad : \quad \sigma_0 = \frac{1}{10\sqrt{5}} .$$

$$= .04472$$

Consider

MINIMUM VARIANCE LINE.

$$w(\mu) = \mu \begin{pmatrix} -5 \\ 0 \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \frac{1}{5}.$$

Let $\bar{w}_0$ be security w/ minimal variance over securities w/ mean $\mu = 0$.

$$\bar{w}_0 = w(\mu = 0) = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \frac{1}{5}.$$

$(\bar{\sigma}_0, \bar{\mu}_0)$ are stddev + mean associated to $\bar{w}_0$.

$$\bar{\mu}_0 = \mu = 0.$$

$$\bar{\sigma}_0^2 = \bar{w}_0^T \Sigma w_0 = \frac{1}{25} \frac{1}{100} (4\ 2\ -1) \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

$$= \frac{30}{2500} = \frac{3}{250}$$

Image in (σ, μ) plane

Consider 2 security market of w_0 + $\bar{w}_0$

Find covariance matrix:

$$\mathrm{Cov}\left(K_{w_0}, K_{\bar{w}_0}\right) = \frac{1}{5000}\begin{pmatrix} 4 & 2 & -1 \end{pmatrix}\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix} = \frac{1}{500}$$

$$\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 6 \end{pmatrix}\frac{1}{500} \qquad \bar{\mu}_0 = 0 \qquad \mu_0 = 1/10$$

$$\mu_V = s\frac{1}{10} \ ; \ \sigma_V^2 = \left\{s^2 + (1-s)^2\, 6 + 2s(1-s)\right\}\frac{1}{500}$$

$$A^2 = \frac{(1+6-2)\frac{1}{500}}{(1/10)^2} = \frac{5}{(1/100)}\left(\frac{1}{500}\right) = 1.$$

$\therefore$ $\mu_V \, \sigma_V$ is hyperbola w/center $(\sigma, \mu) = (0, \frac{1}{10})$
+ asymptotes:

$$\sigma = \frac{1}{10} \pm \mu.$$

2 security subsystems:

system $\underline{\underline{1+2}}$ $\quad \Sigma = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \frac{1}{100}$, $\quad m = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \frac{1}{10}$

$$s_0 = 1/2, \quad \mu_0 = \frac{1}{20}, \quad \sigma_0^2 = \frac{1}{200}$$

$$A^2 = \frac{\sigma_1^2 + \sigma_2^2 - 2c_{12}}{(\mu_1 - \mu_2)} = \frac{(2+2+2)(1/100)}{(1/10)^2} = 6$$

Asymptotes:

$$\mu = \frac{1}{20} \pm \frac{1}{\sqrt{6}} \sigma$$

System 2+3, $\quad \Sigma = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \frac{1}{100} \quad m = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \frac{1}{10}$

$$s_0 = 1/2, \quad \mu_0 = 3/20, \quad \sigma_0^2 = \frac{1}{200}$$

$$A^2 = \frac{6 \frac{1}{100}}{(1/10)^2} = 6$$

Asymptotes:

$$\mu = \frac{3}{20} \pm \frac{1}{\sqrt{6}} \sigma,$$

System 1+3

$$\Sigma = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \frac{1}{100} \quad m = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \frac{1}{10}$$

$$s_0 = 1/2, \quad \mu_0 = 1/10, \quad \sigma_0^2 = \frac{1}{100}$$

$$A^2 = \frac{4 \frac{1}{100}}{(2/10)^2} = 1$$

asymptotes: $\mu = \frac{1}{10} \pm \sigma.$

Add Bond to system.

Suppose $K_B = \frac{1}{50}$

find market portfolio + Capital Market Line.

formula for market portfolio:

$$W_M = \frac{\Sigma^{-1}(m - \mu_B \mathbb{1})}{\mathbb{1}\, \Sigma^{-1}(m - \mu_B \mathbb{1})}$$

$$\cancel{m - \mu_B \mathbb{1} = \begin{pmatrix}0\\1\\2\end{pmatrix}\frac{1}{100} - \begin{pmatrix}1\\1\\1\end{pmatrix}\frac{1}{50} = \begin{pmatrix}1/50\\8/100\\9/50\end{pmatrix} = \begin{pmatrix}2\\8\\18\end{pmatrix}\frac{1}{100}}$$

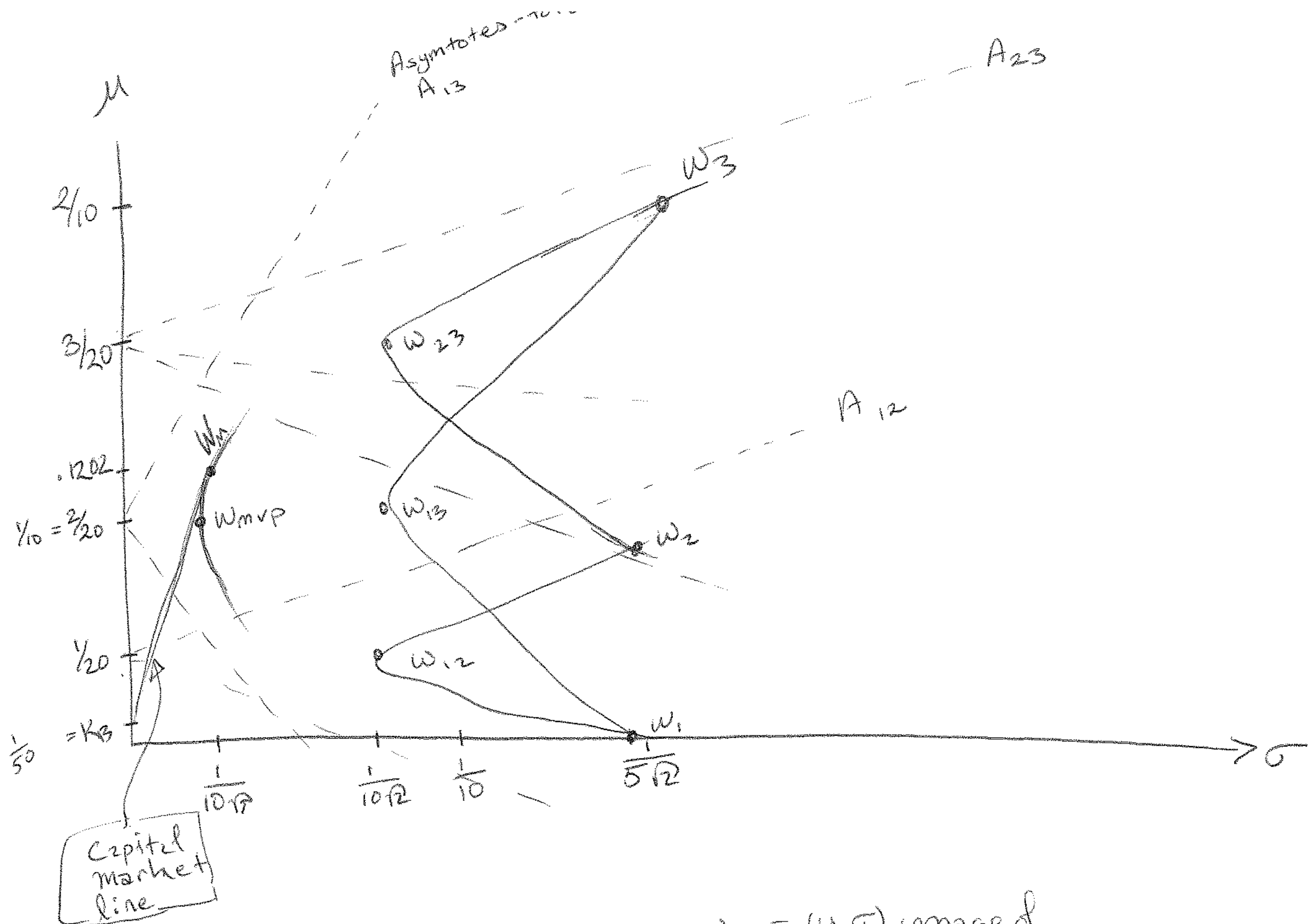
$$\Sigma^{-1} m = \begin{pmatrix}2\\4\\4\end{pmatrix} 5.25 \quad , \quad \mu_B \Sigma^{-1}\mathbb{1} = \begin{pmatrix}6\\8\\6\end{pmatrix}\frac{25}{50} = \begin{pmatrix}3\\4\\3\end{pmatrix}$$

$$W_M = \frac{\begin{pmatrix}2\\4\\4\end{pmatrix}125 - \begin{pmatrix}3\\4\\3\end{pmatrix}}{(250-3)+(500-4)+(500-3)} = \frac{1}{1240}\begin{pmatrix}247\\496\\497\end{pmatrix}$$

$$\hookrightarrow W_M \cong \begin{pmatrix}.199\\.4\\.401\end{pmatrix}$$

Capital market line

$$\mu = \frac{1}{50} + \sigma \sqrt{(m - \mu_B \mathbb{1})^T \Sigma^{-1}\begin{pmatrix}247\\496\\497\end{pmatrix}}$$



$A_{ij} \equiv$ asymptotes for ij subsystem

$w_{ij} \equiv$ mvp for ij subsystem

$w_i \equiv (\mu, \sigma)$ image of i^{th} security

$W_M \equiv$ market portfolio.