Math 458 - Practice Problems for Quiz # 4 - Fall 2023

1. Let $S(t)$ be the price of a stock at time t, modeled via

$$dS(t) = 0.1\, S(t)\, dt + 0.2\, S(t)\, dW(t).$$

Calculate the probability p that after two years, the stock price is larger than its initial price.

2. If $W(t)$ is a standard Brownian motion and

$$X(t) = W(t)^3 + c\, t\, W(t)$$

is a martingale, find c.*

3. Let $W(t)$ be a standard Brownian motion. Compute the quadratic variation of

$$X(t) = 3 + 4W(t)$$

from $t = 0$ to $t = 2$.

4. $\{X(t)\}_{t\geq 0}$ follows arithmetic Brownian motion such that $X(45) = 41$. The drift factor of this Brownian motion is 0.153, and the volatility is 0.98. What is the probability that $X(61) < 50$?

5. For $W(t)$ standard Brownian motion, show that

$$\mathbb{P}(W(1) \leq 0 \quad \text{and} \quad W(2) \leq 0) = \frac{3}{8}.$$

6. Solve the stochastic differential equation

$$dX(t) = \frac{b - X(t)}{1 - t}\, dt + dW(t)$$

where $0 \leq t < 1$ and $X(0) = a$. Here a, $b \in \mathbb{R}$ are arbitrary (but fixed) constants.

***Hint:* Use the fact that an Itô process

$$dX(t) = \mu(X(t), t)\, dt + \sigma(X(t), t)\, dW(t)$$

is a martingale if and only if $\mu(X(t), t) = 0$.