Math 458 - Practice Quiz # 1 - Solution

1. (1 point each) Please circle either T (true) or F (false) for each of the below statements. **ANSWERS ARE IN BOLD.**

a)	**T**	F	A 75-strike European call option is *in the money* if the current asset price is 85.
b)	T	**F**	A 1-year American call option with strike $K = 100$ gives you the right, but not the obligation, to buy the underlying asset for 100 only at the end of each month, up to and including expiration at the end of the year.
c)	**T**	F	If $r = \delta$ for a stock with price S_t, then the forward price $F_{0,T}$ is equal to the asset price S_0.
d)	**T**	F	A short straddle with strike price $K = 100$ and a long strangle with strikes $K_1 = 50$ and $K_2 = 150$ is an example of a long butterfly spread.

2. (6 total points) A stock is priced at time $t = 0$ is $S_0 = \$75$ and pays dividends of \$1 at 3 months and \$3 at 8 months. The annual risk free rate is $r = 5\%$, compounded continuously.

A) (4 points) Find the arbitrage-free forward price $F(0, 1)$ for a 1-year forward.

<u>*Solution:*</u> The prepaid forward price is

$$F^P_{0,T} = S_0 - \text{div}(0,T) = 75 - \left(1 \cdot e^{-0.05(3/12)} - 3 \cdot e^{-0.05(8/12)}\right) \simeq \$71.11.$$

The arbitrage-free forward price for $T = 1$ is thus

$$F_{0,T} = FV\left[F^P_{0,T}\right] = 71.11 \cdot e^{0.05 \cdot 1} \simeq \boxed{\$74.7567.}$$

B) (2 points) If the forward price $F(0, 1)$ is \$75, is there an arbitrage opportunity? If not, explain why not. If so, how much is the arbitrage profit? *If an arbitrage profit does exist, for 2 points of EXTRA CREDIT, explain in detail how you would obtain the arbitrage profit.*

<u>*Solution:*</u> Because the arbitrage-free price of \$74.7567 is not equal to \$75, there must be an arbitrage opportunity. In particular, it is

$$\$75 - \$74.7567 \simeq \boxed{\$0.2433.}$$

3. (5 points) Use a combination of European calls and/or puts with strikes $K_1 < K_2$ and expiration T to create formulae for the payoff $\Lambda(S_T)$ and the profit $\pi(S_T)$ of a European bull spread. Graph your answers on a S_T-Λ, π coordinate system. Assume an annual continuous interest rate of $r > 0$.

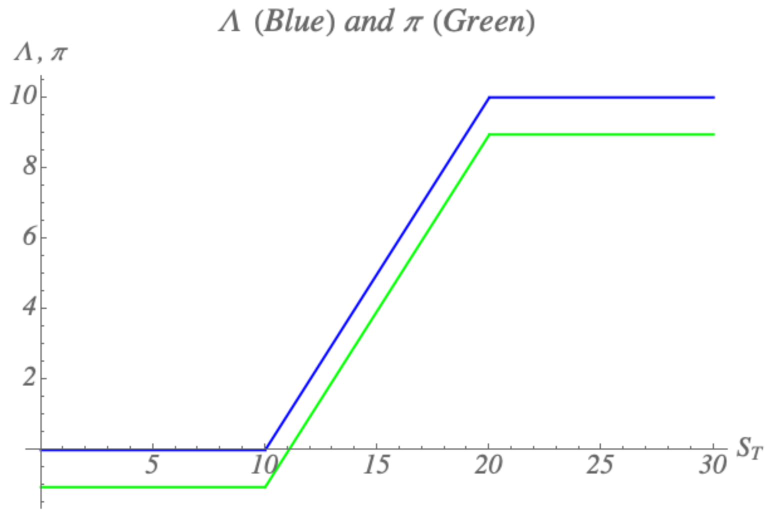
<u>*Solution:*</u> The bull spread can be created by buying a call with strike price K_1 and then selling a call with strike price K_2 with $K_2 > K_1$. Then $C(K_1) > C(K_2)$. The payoff is

$$\Lambda(S_T) = (S_T - K_1)^+ - (S_T - K_2)^+ = \left\{ \begin{array}{ccc} 0 & : & 0 \le S_T \le K_1 \\ S_T - K_1 & : & K_1 < S_T < K_2 \\ K_2 - K_1 & : & S_T \ge K_2 \end{array} \right\}$$

and the profit is

$$\pi(S_T) = \Lambda(S_T) - (C(K_1) - C(K_2))\, e^{rT}.$$

A sample graph is below for the choices $K_1 = 10$, $K_2 = 20$, $r = 5\%$, and $T = 1$:



4. (5 points) A 6-month European call option on a share of MSU stock with strike price of \$35.00 sells for \$2.27. A 6-month European put option on a share of MSU stock with the same strike price sells for $\$P$. The current price of a share is \$32.00. Assuming a 4% continuously compounded risk-free rate and quarterly dividend payments of \$1, starting at 3 months, find P.

   <u>*Solution:*</u> We are given that

   $$T = 1/2, \quad K = 35, \quad C^E = 2.27, \quad S_0 = 32, \quad \text{and} \quad r = 0.04.$$

   The present value of the dividend payments are

   $$\text{div}(0, 1/2) = 1 \cdot e^{-0.04(1/4)} + 1 \cdot e^{-0.04(1/2)} \simeq 1.9702.$$

   Therefore put-call parity states that

   $$C^E - P^E = S_0 - \text{div}(0,1/2) - Ke^{-r/2} \quad \Rightarrow \quad P^E = 2.27 - 32 + 1.97025 + 35e^{-0.02} \simeq \boxed{\$6.5472.}$$