**Math 458 - Financial Math for Actuaries II - Practice Exam # 2 - Fall 2023**

1. Consider a European call option on a stock following the Black-Scholes framework. The option expires in one year. Using the Black-Scholes formula for the option, you obtain

$$N(d_1) = 0.6591 \quad \text{and} \quad N(d_2) = 0.3409.$$

Find the volatility $\sigma$ of the stock.

2. A market-maker sells a 1-year European put option on a non-dividend paying stock and $\Delta$-hedges it. You are given

- The current price of the stock is 50.
- The put option is at-the-money.
- The continuously compounded risk-free interest rate is 8%.
- The volatility of the stock is 22%.
- The option premium is 2.5632.
- The option $\Delta$ is -0.3179.

After 1 week, the stock price is 49. Determine the amount of money required to purchase additional shares to maintain the $\Delta$-hedge.

A) -1.87

B) -1.85

C) -1.83

D) -1.81

E) -1.79

3. A stock has price 55, pays a continuous dividend rate of 2.5%, and has 20% continuous volatility. A portfolio $\Pi$ has two European call options on this stock:

- Call I allows purchase of 100 shares of the stock at the end of 3 months at a strike price of 55;
- Call II allows purchase of 200 shares of the stock at the end of 6 months at a strike price of 60;

If the risk-free continuous rate is 4%, calculate the elasticity of the portfolio.

4. For two options on a non-dividend paying stock following the BSM framework, you are given:

| Option | $\Delta$ | $\Gamma$ | $\theta$ | Option Premium |
|---|---|---|---|---|
| 1 | 0.5600 | 0.0800 | -0.0156 | 3.00 |
| 2 | 0.3000 | 0.0320 | -0.0068 | 1.00 |

In the above table, $\theta$ is expressed *per day* and the stock's price is 50. Determine $r$, the continuously compounded risk-free interest rate.

A) 0.043
B) 0.045
C) 0.048
D) 0.051
E) 0.054

5. A stock's price follows a lognormal distribution. To simulate its price over 10 years, scenarios are generated. In each scenario, the stock price at time $t$ is generated by generating a standard normal random variable $Z$. Then $S_t = S_{t-1}e^{0.1+0.2\,Z}$ for $t = 1, 2, \ldots, 10$. Find the expected value of the ratio $S_{10}/S_0$ in this simulation.

6. You are given:

- A stock has price 45.
- A market-maker writes put option I on a stock with price 1.53, $\Delta = -0.39$, and $\Gamma = 0.072$.
- The market-maker $\Delta$-$\Gamma$ hedges the option with the stock and with put option II having price 2.00, $\Delta = -0.31$, and $\Gamma = 0.038$.

Determine the number of shares of stock to buy to implement the hedge.

A) 0.19
B) 0.20
C) 0.21
D) 0.22
E) 0.23

7. Let $S_t$ be the price of a stock at time $t$. $S_t$ follows a lognormal model and you are given the following information:

- The continuously compounded expected annual rate of return on the stock is 15%.
- The continuously compounded dividend rate is 2%.
- The stock's volatility is 30%.
- $S_0 = 80$.

Calculate the probability that a 4-year European call with strike $K = 130$ has a payoff of 20 at expiration.