

Math 313 - Topics in Actuarial Mathematics - Practice Exam # 1

1. (20 total points - 2 points each) Please circle either T (true) or F (false) for each of the below statements. All prices are no-arbitrage prices.

I) T F Combining a short straddle with strike price $K_2 = \$60$ with a long strangle with strikes $K_1 = 30$ and $K_3 = 90$ forms a symmetric butterfly spread.

II) T F If $r = \delta$ for a stock with price S_t , then the forward price $F_{0,T}$ is equal to the asset price S_0 .

III) T F It is never advantageous to exercise an American call option early.

IV) T F Given 1-year European puts with risk-free rate is $r = 10\%$, $\delta = 5\%$, and $\sigma = 20\%$, it is possible for the arbitrage-free price $P^E(K)$ to satisfy

$$P^E(30) > P^E(10) + 20e^{0.25}.$$

V) T F In the Cox-Ross-Rubenstein binomial model, if $u = 1.2$ then $d = 0.8$.

VI) T F In a 5-period binomial tree with initial stock price $S_0 = \$100$, expiration $T = 1$ year, $u = 1.1$, and $d = 0.9$, the second highest value of S_1 is \$131.769.

VII) T F Given a risk-free rate $r > 0$ and continuous dividend rate $\delta > 0$, one way to compute the risk-neutral probability \tilde{p} for the binomial model is to require that the risk-neutral expected value of the stock S_{t+1} is equal to the forward price of the stock. That is, solve

$$\tilde{E}[S_{t+1}] = \tilde{p}(uS_t) + (1 - \tilde{p})(dS_t) = S_te^{(r-\delta)h}.$$

for \tilde{p} .

VIII) T F In a 3-period binomial tree where the probability that the stock moves “up” is p , the probability that the stock has two “up” moves and one “down” move, in any order, is $p^2(1 - p)$.

IX) T F Because the Black-Scholes formula gives the arbitrage-free price of a call option in continuous time, you cannot use it to price a call on a stock where only discrete dividends are paid (e.g., owning a stock pays dividends only twice/year).

X) T F For a 6-month European option, modeled using the Black-Scholes-Merton framework, $d_1 = d_2$ requires $\sigma = 0$.

2. (20 points) Consider a European call option and a European put option on a non-dividend paying stock. The current stock price is \$60, the call option currently sells for \$0.15 more than the put option. Both the call and put option have strike price $K = \$70$ and will expire in 4 years. Find the continuously compounded annual risk-free interest rate r .

3. (20 points) It is known that for a European put option, the arbitrage-free price $P^E(K)$ satisfies

$$P^E(20) = 55 \quad \text{and} \quad P^E(110) = 125.$$

A) (14 points) Use convexity to find a minimum value of $P(155)$.

B) (6 points) Find an upper bound to $P(155)$ and compare it to your answer in (A).

4. (20 total points) MSU stock is currently worth \$60 a share. The continuously compounded risk-free rate is $r = 6\%$, the annual continuously compounded dividend yield is $\delta = 2\%$, and the annualized standard deviation of the continuously compounded stock return is $\sigma = 15\%$.

A) (6 points) Using the Cox-Ross-Rubenstein model, find up and down factors u and d for a 2-period binomial model over the course of one year.

B) (4 points) Use your answer to part (A) to draw the 2-period stock price tree.

C) (10 points) Use your answers to parts (A) & (B) to find the arbitrage free price of a 1-year European derivative with payoff at expiration given by

$$\Lambda(S) := \left\{ \begin{array}{lll} 0 & : & S < 55 \\ 10 & : & 55 \leq S \leq 70 \\ 0 & : & S > 70 \end{array} \right\}.$$

5. (20 points) You are considering the purchase of 100 units of a 3-month 25-strike European put option on a stock. You know that

- The Black-Scholes framework holds.
- The stock currently sells for \$20.
- The stock's volatility is 24%.
- The stock pays dividends continuously with a dividend yield of 3%.
- The continuously compounded risk-free interest rate is 5%.

Find the price of a block of 100 of these put options. Be sure to show **ALL** of your work.

6. (20 points) A stock price S_t that follows the BSM framework, you know $S_0 = 40$. Further, you are given

- The continuously compounded expected rate of return on the stock is 12%.
- The continuously compounded dividend yield is 2%.
- The continuously compounded risk-free interest rate is 3%.

European call and put options expiring in one year have strike price 50. The expected payoff on the call option is 4.40. Find the expected payoff of the put option.

- I) 9.89
- II) 10.19
- III) 11.95
- IV) 13.71
- V) 14.40