

Do any 7 of the following 10 exercises of your choice. Write up your solutions neatly in your own handwriting, and show all your work!

1. Do problem 8 on page 320 of Folland. You'll want to use Theorem 9.3 on page 315.
2. Do problem 9 on page 320 of Folland. Start by proving that  $k^{-1} \sin(kx) \rightarrow 0$  weakly as  $k \rightarrow \infty$ , and then use Theorem 9.3 on page 315 together with integration by parts (recall page 308).
3. Do problem 11 on page 320 of Folland with  $n = 1$ . In this case partial differentiation becomes standard MTH132 differentiation with respect to a single variable.
4. Do problem 1 on page 327 of Folland. Note: The function  $\chi$  has a typo in its definition. It should be

$$\int_{|x|}^{2\pi} e^{-1/t(2\pi-t)} dt$$

divided by  $\int_0^{2\pi} e^{-1/t(2\pi-t)} dt$  for all  $|x| < 2\pi$ . In general, the  $1-t$  terms in the exponents and hint for part *a* should be  $2\pi - t$  terms.

5. Do problem 2 on page 328 of Folland.
6. Find the Fourier series of the second derivative of entry 10 in Table 1 on page 26 of Folland. Hint: See problem 2 (b) on page 328 of Folland!
7. Prove that (a)  $(F * \delta) = F$  for all distributions  $F$ , and that (b)  $\widehat{\delta}_c = e^{-icx}$  for all shifts  $c \in \mathbb{R}$ . Make sure to show every step for both parts, to unwind the notation very carefully, and to reference the correct part of Theorem 7.5 on page 214 when necessary.
8. Compute (a)  $\widehat{\cos(x)}$ , (b)  $\widehat{\sin(3x)}$ , and (c)  $\widehat{|\sin x|}$ . Take a look at example 5 on page 337 of Folland before you start, and refer to the table on page 26 of Folland for part (c).
9. Do problem 6 on page 340 of Folland.
10. Do problem 7 on page 340 of Folland.

**You are now a Fourier master: Go forth and transform the world!**