Do any 7 of the following 10 exercises of your choice. Write up your solutions neatly in your own handwriting! I recommend problems 9 and 10 for students who want a deeper understanding of the underlying theoretical issues involving Fourier series. Most problems are from the book, which is available online here:
`http://www-elec.inaoep.mx/~rogerio/FourierAnalysisUno.pdf`.

1. Verify formula 7 in Table 1 on page 26 of Folland.

2. Verify formula 8 in Table 1 on page 26 of Folland.

3. Verify formula 19 in Table 1 on page 28 of Folland.

4. Use formula 8 in Table 1 of Folland to derive formula's 9 and 10 in Table 1 (indirectly).

5. Do Exercise 1 on page 37 of Folland.

6. Do Exercise 2 on page 37 of Folland.

7. Do Exercises 3, 4, and 5 on page 37 of Folland.

8. Do Exercises 6 and 7 on page 37 of Folland.

9. Consider the function $f(x) = \sin(1/x)$ on $[-\pi, \pi]$, where we define $f(0) = 0$.
   (a) Sketch this rather strange function. Is it Riemann integrable? Why, or why not?
   (b) Prove that it's Fourier coefficients c_n approach zero as $n \to \infty$.

10. Now consider the function $f(x) = 1/|x|^{\frac{1}{4}}$ on $[-\pi, \pi]$, where we again define $f(0) = 0$. Bessel's Inequality on page 30 of Folland does not directly apply to this function (since it is unbounded it is only integrable as an improper integral, i.e., as the limit of integrals of Riemann integrable functions, and so is not Riemann integrable in its own right). We will have a general theory for functions like this later in the course. For now, we can prove a few things about this specific f.
    (a) Prove that every Fourier coefficient, c_n, of f exists.
    (b) Do the Fourier coefficients exist for its derivative, df/dx, (again with df/dx set to 0 at 0)?
    (c) Suppose that $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$ for all $x \in (0, 1)$. Does $\sum_{n=-\infty}^{\infty} |c_n|$ converge?
    (d) Based on what we know so far, can we claim that the Fourier coefficients of f, c_n, do not approach zero as $n \to \infty$. Why, or why not?