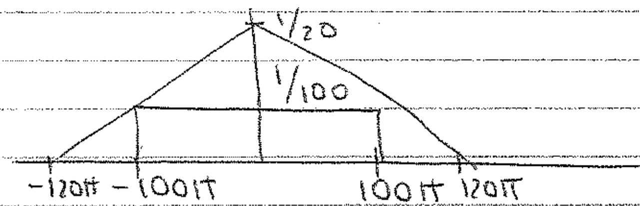
HW 9

① 8.1-2

(c) $\operatorname{sinc}(100\pi t) + 3\operatorname{sinc}^2(60\pi t)$

$$X(\omega) = \frac{1}{100}\operatorname{rect}\left(\frac{\omega}{200\pi}\right) + \frac{1}{20}\Delta\left(\frac{\omega}{240\pi}\right)$$



$BW = 120\pi \qquad \omega_s \geq 240\pi$ rad/sec

(d) $\operatorname{sinc}(50\pi t)\operatorname{sinc}(100\pi t) \xleftrightarrow{\ \mathcal{F}\ } \frac{1}{2\pi}\left(\frac{1}{50}\operatorname{rect}\left(\frac{\omega}{100\pi}\right) * \frac{1}{100}\operatorname{rect}\left(\frac{\omega}{200\pi}\right)\right)$

The convolved signal will be a triangle and will go from -150π to 150π in ω.

$\omega_s \geq 300\pi$ rad/sec.

② 8.1-7

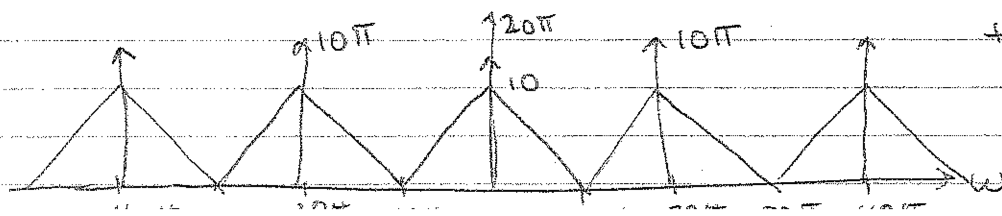
a) $x(t) = 5\operatorname{sinc}^2(5\pi t) + \cos(20\pi t)$

$$X(\omega) = \Delta\left(\frac{\omega}{20\pi}\right) + \pi[\delta(\omega - 20\pi) + \delta(\omega + 20\pi)]$$

$\omega_s = 20\pi$ rad/sec $\qquad \frac{1}{T} = 10$

$$X_s(\omega) = 10\sum_{k=-\infty}^{\infty} X(\omega - k20\pi)$$

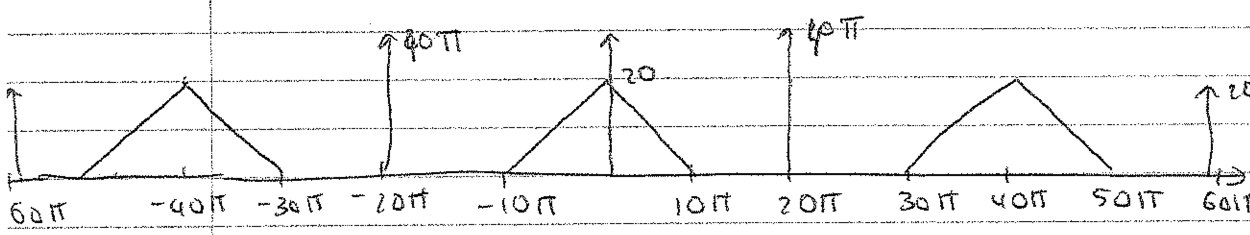
$$= 10\sum_{k=-\infty}^{\infty} \Delta\left(\frac{\omega - k20\pi}{20\pi}\right) + \pi[\delta(\omega - k20\pi - 20\pi) + \delta(\omega - k20\pi + 20\pi$$



we cannot reconstruct $x(t)$ by LPF.

b) $\omega_s = 40\pi \text{ rad/sec} \quad T = \frac{1}{20}$

$X_s(\omega) = 20 \sum X(\omega - k\,40\pi)$



with LPF cutoff 20π rad/sec or 10Hz and gain of $1/20$, the output will be

$\text{tri}\left(\frac{\omega}{20\pi}\right) + \pi[\delta(\omega - 20\pi) + \delta(\omega + 20\pi)]$ and

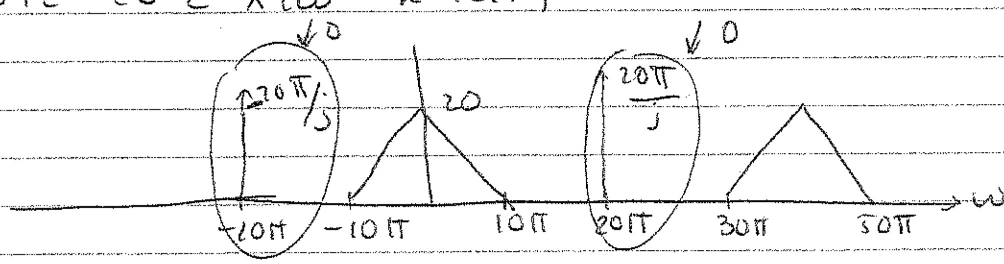
we can get back the original $x(t)$.

Note: we assume that the gain of the filter at cutoff is $1/40$ since the rect function value is 0.5 at the edges.

c) $x(t) = 5\,\text{sinc}^2(5\pi t) + \sin(20\pi t)$

$\hookrightarrow X(\omega) = \text{tri}\left(\frac{\omega}{20\pi}\right) + \frac{\pi}{j}\left[\delta(\omega - 20\pi) - \delta(\omega + 20\pi)\right]$

$X_s(\omega) = 20 \sum X(\omega - k\,40\pi)$

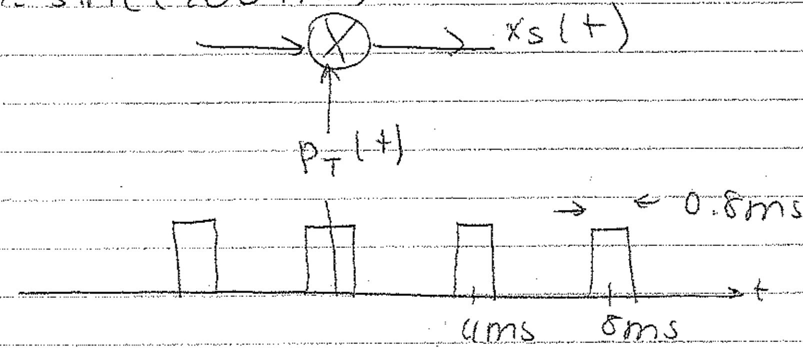


The impulses will overlap with opposite signs and will cancel, so there's no way to recover the sine component.

d) Yes, no cancellation

③ 8.2-1

$x(t) = \mathrm{sinc}(200\pi t)$



$$X_s(\omega) = \frac{1}{2\pi} X(\omega) * P_T(\omega)$$

$$\frac{1}{2\pi}\left(\frac{1}{200}\,\mathrm{rect}\left(\frac{\omega}{400\pi}\right)\right) * P_T(\omega)$$

$$P_T(\omega) = \omega_0 \sum_{k=-\infty}^{\infty} G(k\omega_0)\,\delta(\omega - k\omega_0)$$

$$g(t) = \mathrm{rect}\left(\frac{t}{0.8\times10^{-3}}\right) \longleftrightarrow G(\omega) = 0.8\times10^{-3}\,\mathrm{sinc}\left(0.4\times10^{-3}\omega\right)$$

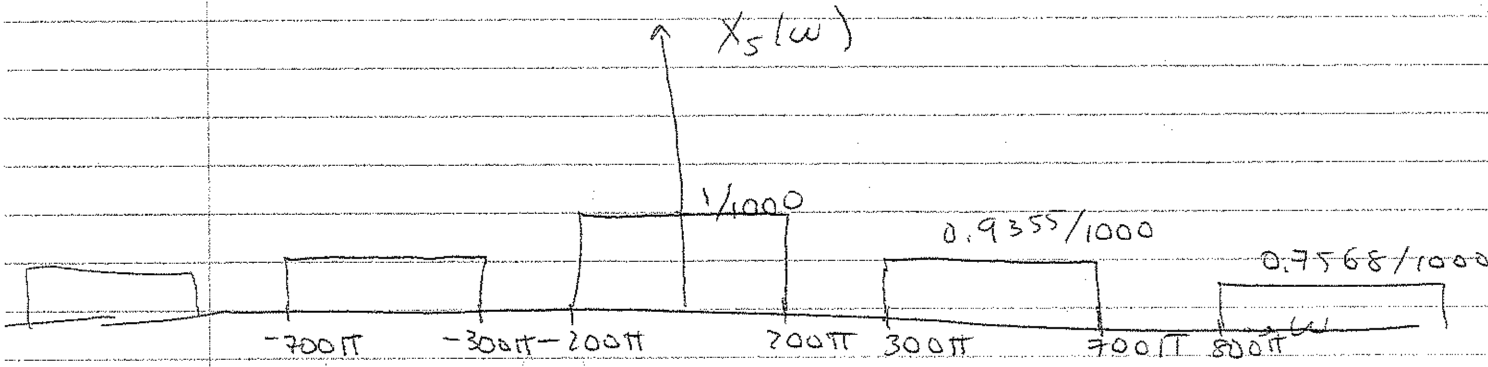
$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{4\times10^{-3}} = 500\pi$$

$$P_T(\omega) = 500\pi \sum_{k=-\infty}^{\infty} (0.8\times10^{-3})\,\mathrm{sinc}\left(0.4\times10^{-3}(500\pi)k\right)\delta(\omega - 500\pi k)$$

$$= 0.4\pi \sum_{k=-\infty}^{\infty} \mathrm{sinc}(0.2\pi k)\,\delta(\omega - 500\pi k)$$

$$X_s(\omega) = \frac{0.4\cancel{\pi}}{400\cancel{\pi}} \sum_{k=-\infty}^{\infty} \mathrm{sinc}(0.2\pi k)\,\mathrm{rect}\left(\frac{\omega - 500\pi k}{400\pi}\right)$$

$$= \frac{1}{1000} \sum_{k=-\infty}^{\infty} \mathrm{sinc}(0.2\pi k)\,\mathrm{rect}\left(\frac{\omega - 500\pi k}{400\pi}\right)$$



$k=1$, $\text{sinc}(0.2\pi) = \dfrac{\sin(0.2\pi)}{0.2\pi} = 0.9355$

if passed through LPF with BW = 100 Hz → 200π rad/s

The output $\dfrac{1}{1000}\,\text{rect}\left(\dfrac{\omega}{400\pi}\right)$

if passed through LPF with cutoff between 100 Hz & 150 Hz → $\dfrac{1}{1000}\,\text{rect}\left(\dfrac{\omega}{400\pi}\right)$

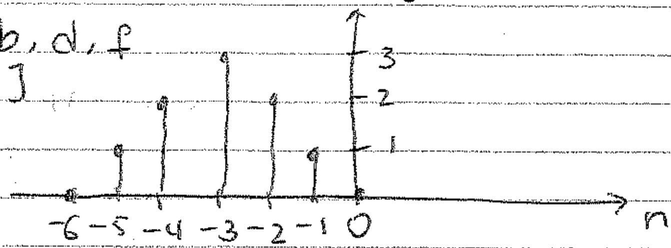
if cutoff > 150 Hz (300π rad/sec) → aliasing

ECE 366 HW#7

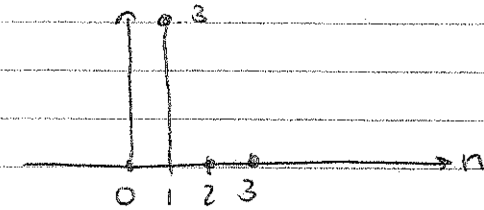
Solutions

4) 3.2-3 b, d, f

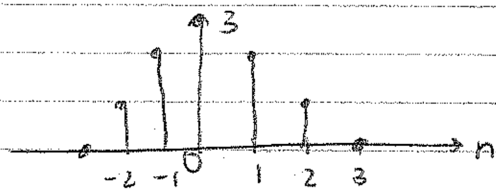
b) $x[n+6]$



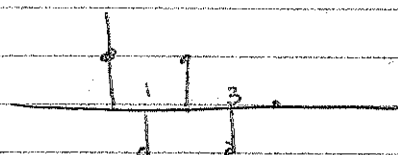
d) $x[3n]$



f) $x[3-n]$



5) a) $(-1)^n u[n]$



power signal

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1 = \lim_{N\to\infty} \frac{N+1}{2N+1}$$

$$= 1/2$$

b) Energy signal

$$E_x = 2[9+36+81] = 2[126] = 252$$

c) $\cos\left[\frac{\pi}{3}n + \frac{\pi}{6}\right]$ periodic

$$\Omega_0 = \frac{\pi}{3} = \left(\frac{k}{N_0}\right)2\pi = \left(\frac{1}{6}\right)2\pi$$

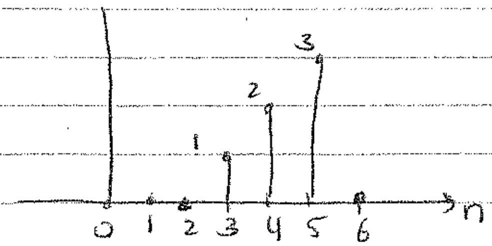
$N_0 = 6$

$$P = \frac{1}{6}\sum_{n=0}^{5}\left(\cos^2\left(\frac{\pi}{6}\right) + \cos^2\left(\frac{\pi}{2}\right) + \cos^2\left(\frac{5\pi}{6}\right) + \cos^2\left(\frac{7\pi}{6}\right) + \cos^2\left(\frac{3\pi}{2}\right) + \cos^2\left(\frac{11\pi}{6}\right)\right]$$

$$= \frac{1}{6}\left[\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}\right] = \frac{1}{6}\left[\frac{12}{4}\right] = \frac{1}{2}$$

6) 3,3 - 3 (c), (e)

c) $n-2 \{u[n-2] - u[n-6]\}$



e) $(n-2)\{u[n-2] - u[n-6]\} + (-n+8)\{u[n-6] - u[n-9]\}$