HW 6 Solns

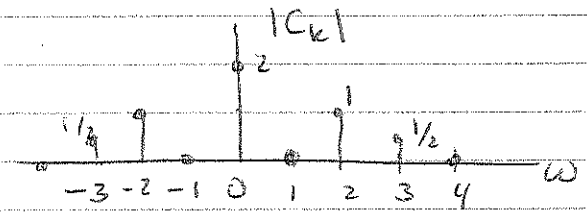
1. 6.3-6 (a) (b)

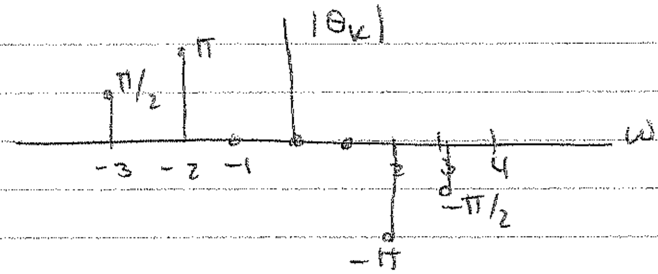
$\omega_0 = 1$ rad/sec

a) $x(t) = 2 + 2\cos(2t - \pi) + \cos(3t - \pi/2)$

$= 2 - 2\cos(2t) + \cos(3t - \pi/2)$

b)





only the second and third harmonics exist

2 a) $x(t) = e^{at}[u(t) - u(t-T)]$

$$X(\omega) = \int_0^T e^{at} e^{-j\omega t} dt$$

$$= \int_0^T e^{(a-j\omega)t} dt$$

$$= \left. \frac{e^{(a-j\omega)t}}{a-j\omega} \right|_0^T = \frac{e^{(a-j\omega)T} - 1}{a-j\omega}$$

b) $x(t) = \text{rect}\left(\frac{t-10}{8}\right)$

$$\text{rect}\left(\frac{t}{8}\right) \xleftrightarrow{\mathcal{F}} 8\,\text{sinc}(4\omega)$$

$$\text{rect}\left(\frac{t-10}{8}\right) \xleftrightarrow{\mathcal{F}} 8e^{-j10\omega}\,\text{sinc}(4\omega)$$

c) $\delta(t+2) - \delta(t-2) \xleftrightarrow{\mathcal{F}} e^{j2\omega} - e^{-j2\omega}$

$$= 2j\sin(2\omega)$$

d) $x(t) = \cos(t)\,\text{rect}\left(\frac{t - \pi/4}{\pi/2}\right)$

$$X(\omega) = \frac{1}{2\cancel{\pi}}\left[\cancel{\pi}(\delta(\omega-1) + \delta(\omega+1)) * \frac{\pi}{2} e^{-j\pi/4\,\omega}\,\text{sinc}\left(\frac{\pi\omega}{4}\right)\right]$$

$$= \frac{\pi}{4}\left[e^{-j\frac{\pi}{4}(\omega-1)}\,\text{sinc}\left(\frac{\pi(\omega-1)}{4}\right) + e^{-j\pi/4(\omega+1)}\,\text{sinc}\left(\frac{\pi(\omega+1)}{4}\right.\right.$$

e) $2[u(t) - u(t-6)] \xleftrightarrow{\mathcal{F}} 2\pi\delta(\omega) + \frac{2}{j\omega} - 2e^{-j6\omega}\left(\pi\delta(\omega) + \frac{}{j\omega}\right.$

$$= \frac{2}{j\omega} - \frac{2e^{-j6\omega}}{j\omega}$$

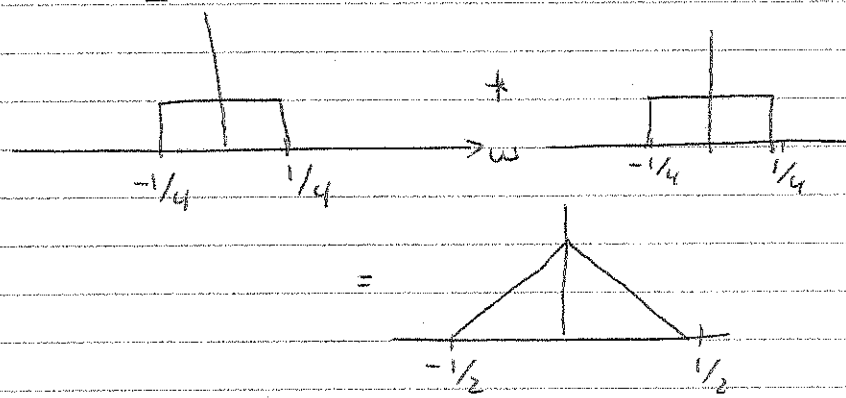
$$= 2e^{-j3\omega}\left[\frac{e^{j3\omega} - e^{-j3\omega}}{}\right] = 4e^{-j3\omega}\,\frac{\sin(3\omega)}{}$$

$12 e^{-j3\omega} \operatorname{sinc}(3\omega)$

f) $4\operatorname{sinc}^2(t/4) \xrightarrow{\mathcal{F}} 16\pi\, \Delta\left(\frac{\omega}{\,}\right)$ From the table.

OR. $4\operatorname{sinc}^2(t/4) = \left(2\operatorname{sinc}(t/4)\right)^2 \xleftrightarrow{\mathcal{F}}$

$$\frac{1}{2\pi}\left[8\pi \operatorname{rect}(2\omega) * 8\pi \operatorname{rect}(2\omega)\right]$$



3) a) $X(\omega) = \operatorname{rect}\left(\frac{\omega}{2\omega_0}\right) e^{-j\omega t_0}$

$$x(t) = \frac{\omega_0}{\pi} \operatorname{sinc}\left(\omega_0 (t - t_0)\right)$$

b) $X(\omega) = \operatorname{rect}\left(\frac{\omega - 10}{2\pi}\right)$

$x(t) = \operatorname{sinc}(\pi t)\, e^{j10t}$ (freq. shift property)

c) $X(\omega) = \cos(\omega) \operatorname{rect}\left(\frac{\omega}{\pi}\right)$

$$x(t) = \mathcal{F}^{-1}(\cos(\omega)) * \mathcal{F}^{-1}\left(\operatorname{rect}\left(\frac{\omega}{\pi}\right)\right)$$

$$\downarrow$$

$$* \ \frac{1}{2}\operatorname{sinc}\left(\frac{\pi t}{2}\right)$$

$$\cos(t) \xleftrightarrow{\mathcal{F}} \pi[\delta(\omega-1) + \delta(\omega+1)]$$

by duality

$$\pi[\delta(t-1) + \delta(t+1)] \xleftrightarrow{\mathcal{F}} 2\pi\cos(-\omega)$$

$$= 2\pi\cos(\omega)$$

$$\frac{1}{2}[\delta(t-1) + \delta(t+1)] \longleftrightarrow \cos(\omega)$$

$$x(t) = \frac{1}{2}[\delta(t-1) + \delta(t+1)] * \frac{1}{2}\operatorname{sinc}\left(\frac{\pi t}{2}\right)$$

$$= \frac{1}{4}\operatorname{sinc}\left(\frac{\pi(t-1)}{2}\right) + \frac{1}{4}\operatorname{sinc}\left(\frac{\pi(t+1)}{2}\right)$$

d) $X(\omega) = \omega^2 \operatorname{rect}\left(\frac{\omega}{2\omega_0}\right)$

$$x(t) = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} \omega^2 e^{j\omega t}\, d\omega \quad \text{OR}$$

$$x(t) = \mathcal{F}^{-1}(\omega^2) * \mathcal{F}^{-1}\left(\operatorname{rect}\left(\frac{\omega}{2\omega_0}\right)\right)$$

$$\frac{\omega_0}{\pi}\operatorname{sinc}(\omega_0 t)$$

let $G(\omega) = 1$ $\quad X(\omega) = -(j\omega)^2 G(\omega)$

$g(t) = \delta(t)$

$$\frac{d^2 g(t)}{dt^2} \xleftrightarrow{\mathcal{F}} (j\omega)^2 G(\omega)$$

$$\frac{d^2\delta(t)}{dt^2} \xleftrightarrow{\mathcal{F}} -\omega^2$$

$$-\frac{d^2\delta(t)}{dt^2} \xleftrightarrow{\mathcal{F}} \omega^2$$

$$\int \omega^2 e^{j\omega t}\, d\omega = \frac{e^{j\omega t}}{(jt)^3}\left[(jt)^2\omega^2 - 2jt\omega + 2\right]\Bigg|_{-\omega_0}^{\omega_0}$$

$$= \frac{(\omega_0^2 t^2 - 2)\sin \omega_0 t + 2\omega_0 t \cos \omega_0 t}{\pi t^3}$$

e) $$X(\omega) = \Delta\left(\frac{\omega - 4}{4}\right) + \Delta\left(\frac{\omega + 4}{4}\right)$$

$$x(t) = \cos(4t)\left[\frac{1}{\pi}\,\mathrm{sinc}^2(t)\right]$$ modulation property