# ECE HW #5

1) a) $h(t) = u(t+1)$

- dynamic
- non-causal
- unstable $\int_{-\infty}^{\infty} |u(t+1)| dt = \int_{-1}^{\infty} 1 dt \to t \Big|_{-1}^{\infty} \to \infty$

b) $h(t) = u(t+2) - u(t-2)$

- dynamic
- non causal

$\int |h(t)| dt = \int_{-\infty} |u(t+2) - u(t+2)| dt$

$= \int_{-2}^{2} 1 \, dt = 4 < \infty \to$ stable

c) $h(t) = e^{-2t} u(t-1)$

- dynamic
- causal $h(t) = 0$ for $t < 0$

$\int_{-\infty}^{\infty} |h(t)| dt = \int_{1}^{\infty} e^{-2t} dt = \frac{e^{-2t}}{-2} \Big|_{1}^{\infty}$

$= \frac{e^{-2}}{2} < \infty$ stable

d) $h(t) = e^{t} \sin(5t) u(t)$

- dynamic
- causal

$\int_{-\infty}^{\infty} |h(t)| dt = \int^{\infty} e^{t} |\sin(5t)| dt \le \int_{0}^{\infty} e^{t} dt \to \infty$

unstable

(2) $H(s) = \dfrac{2s+10}{s^2+2s+10}$

a) $x(t) = 4$ , $s = 0$ $\quad y(t) = H(0)\, 4$

$$= \frac{10}{10}\, 4 = 4$$

b) $x(t) = e^{-2t}$ , $s = -2$ $\quad y(t) = H(-2)\, e^{-2t}$

$$= \frac{6}{10}\, e^{-2t} = \frac{3}{5}\, e^{-2t}$$

c) $x(t) = 4\sin(3t)$ ; $s = 3j$

$$y(t) = 4\,|H(3j)|\,\sin(3t + \angle H(3j))$$

$H(3j) = \dfrac{6j+10}{1+6j} \rightarrow |H(3j)| = \dfrac{\sqrt{136}}{\sqrt{37}} = 1.9172$

$$\angle H(3j) = \tan^{-1}\left(\frac{6}{10}\right) - \tan^{-1}(6)$$

$$= \tan^{-1}(3/5) - \tan^{-1}(6)$$

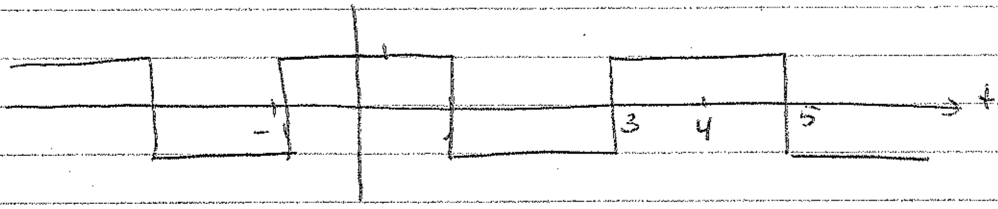
$y(t) = 7.67 \sin(3t - 49.57°)$

d) $x(t) = 4e^{j3t}$ $\quad y(t) = 4\,H(3j)\,e^{j3t}$

$$= 7.67\, e^{j(3t - 49.57°)}$$

e) $y(t)$ in part (c) is the imaginary part of $y(t)$ in part (d).

4 a)




$T_0 = 4$

$\omega_0 = \pi/2$ $\qquad\qquad C_0 = 0.$

$$C_k = \frac{1}{4}\int_0^4 x(t)\, e^{-jk\frac{\pi}{2}t}\,dt$$

$$= \frac{1}{4}\left[\int_0^1 e^{-jk\pi/2\,t}\,dt + \int_1^3 -e^{-jk\frac{\pi}{2}t}\,dt + \int_3^4 e^{-jk\frac{\pi}{2}t}\,dt\right]$$

$$= \frac{1}{4}\left[\left.\frac{e^{-jk\pi/2\,t}}{-jk\pi/2}\right|_0^1 + \left.\frac{e^{-jk\frac{\pi}{2}t}}{jk\pi/2}\right|_1^3 + \left.\frac{e^{-jk\pi/2\,t}}{-jk\pi/2}\right|_3^4\right]$$

$$= 1/4\left[\frac{e^{-jk\pi/2}}{-jk\pi/2} + \frac{1}{jk\pi/2} + \frac{e^{-jk3\pi/2}}{jk\pi/2} - \frac{e^{-jk\pi/2}}{jk\pi/2} + \frac{\overbrace{e^{-jk2\pi}}^{=1}}{-jk\pi/2}\right.$$

$$\left. + \frac{e^{-jk3\pi/2}}{jk\pi/2}\right]$$

$$= \frac{1}{4jk\pi/2}\left[-e^{-jk\pi/2} + 1 + \cancel{e^{-jk3\pi/2}} - e^{-jk\pi/2} - 1 + \cancel{e^{-jk3\pi/2}}\right]$$

$$= \frac{1}{jk2\pi}\left[2e^{-jk\frac{3\pi}{2}} - 2e^{-jk\pi/2}\right] = \frac{2e^{-jk\pi}}{jk2\pi}\left[e^{-jk\pi/2} - e^{+jk\pi/2}\right]$$

$$\frac{-2e^{-jk\pi}}{jk2\pi}\left[e^{jk\pi/2} - e^{-jk\pi/2}\right]$$

$\downarrow$

$$\frac{-4j\,e^{-jk\pi}\sin\left(\frac{\pi}{2}k\right)}{jk2\pi}$$

$$= \frac{-2e^{-jk\pi}}{\pi k}\sin\left(\frac{\pi}{2}k\right)$$

$jk\pi$

#3.

```
>> Numerator=[2,10];
>> Denominator=[1,2,10];
>> polyval(Numerator,0)/polyval(Denominator,0)

ans =

     1

>> polyval(Numerator,-2)/polyval(Denominator,-2)

ans =

    0.6000

>> polyval(Numerator,3i)/polyval(Denominator,3i)

ans =

   1.2432 - 1.4595i

>> abs(polyval(Numerator,3i)/polyval(Denominator,3i))

ans =

    1.9172

>> angle(polyval(Numerator,3i)/polyval(Denominator,3i))

ans =

   -0.8652

>> ans*(360/(2*pi))

ans =

  -49.5739

>>
```

→ $H(0) = 1$

→ $H(-2) = .6 = \frac{3}{5}$

→ $H(3i) \approx 1.2432 - 1.4595i$

→ magnitute $|H(3i)| = 1.9172$

→ phase of $H(3i)$ in both radians & degrees

→ Simply wanted you to use polyval to help you check the values of

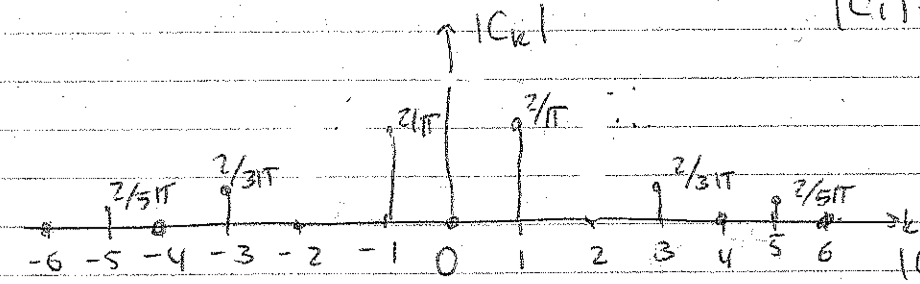
$H(0)$, $H(-2)$, & $H(3i)$

$$|C_k| = \left|\operatorname{sinc}\left(\frac{\pi}{2}k\right)\right|$$

$$\angle C_k = \theta_k = \text{angle}\left\{e^{j\pi}e^{-jk\pi}\operatorname{sinc}\left(\frac{\pi}{2}k\right)\right\}$$

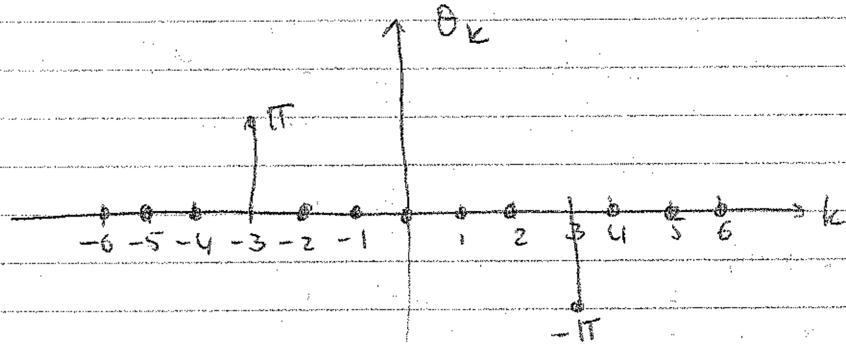
$$= (\pi - k\pi) \pm \pi / \pm 0 .$$

$$|C_1| = \left|\operatorname{sinc}\left(\frac{\pi}{2}\right)\right| = \frac{1}{\pi/2} = 2/\pi$$




$$|C_3| = \left|\operatorname{sinc}\left(\frac{3\pi}{2}\right)\right| = \frac{1}{3\pi/2} = \frac{2}{3\pi}$$

$$|C_5| = 1/_{5\pi/2} = 2/_{5\pi}$$




$$C_1 = -e^{-j\pi}\operatorname{sinc}(\pi/2) \qquad \angle C_1 = 0$$
$$= \operatorname{sinc}(\pi/2)$$

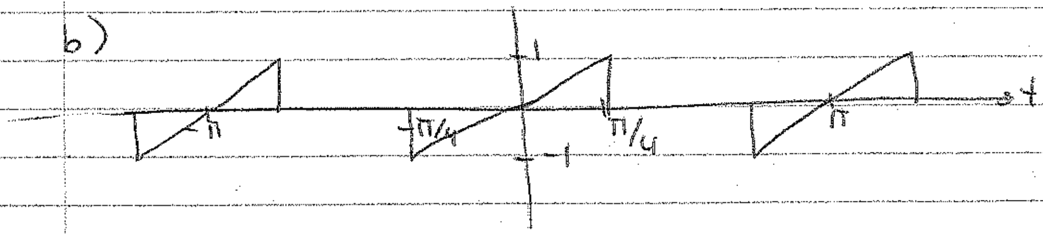
$$C_3 = e^{-j\pi}e^{-j3\pi}\operatorname{sinc}\left(\frac{3\pi}{2}\right) = \operatorname{sinc}(3\pi/2) \qquad \angle C_3 = -\pi$$

$$C_5 = e^{-j\pi}e^{-j5\pi}\operatorname{sinc}(5\pi/2) = \operatorname{sinc}(5\pi/2) \qquad \angle C_5 = 0$$

$$A_k = 2|C_k|\cos(\theta_k) = 2\left|\operatorname{sinc}\left(\pi/_2\, k\right)\right|\cos(\underbrace{\theta_k}_{0 \text{ or } \pm\pi})$$
$$B_k = 0$$
$$= \pm 2\left|\operatorname{sinc}(\pi/_2\, k)\right|$$

even symmetric $\rightarrow B_k = 0$

b)




$T_0 = \pi \qquad \omega_0 = 2 \qquad\qquad C_0 = 0$

$$C_k = \frac{1}{\pi}\int_{-\pi/4}^{\pi/4} \underbrace{x(t)}_{\frac{4}{\pi}t} e^{-jk2t}\,dt$$

$$= \frac{4}{\pi^2}\int_{-\pi/4}^{\pi/4} t\, e^{-jk2t}\,dt$$

Integration by parts: $\dfrac{t\,e^{-jk2t}}{-jk2} + \displaystyle\int \frac{e^{-jk2t}}{jk2}\,dt$

$$\left.\frac{te^{-jk2t}}{-jk2} + \frac{e^{-jk2t}}{k^2 4}\right|_{-\pi/4}^{\pi/4}$$

$$= \frac{4}{\pi^2}\left[\frac{\pi}{4}\,\frac{e^{-jk\pi/2}}{-jk2} + \frac{e^{-j\pi/2 k}}{4k^2} + \frac{-\pi/4\, e^{+jk\pi/2}}{jk2} - \frac{e^{+jk\pi/2}}{k^2 4}\right]$$

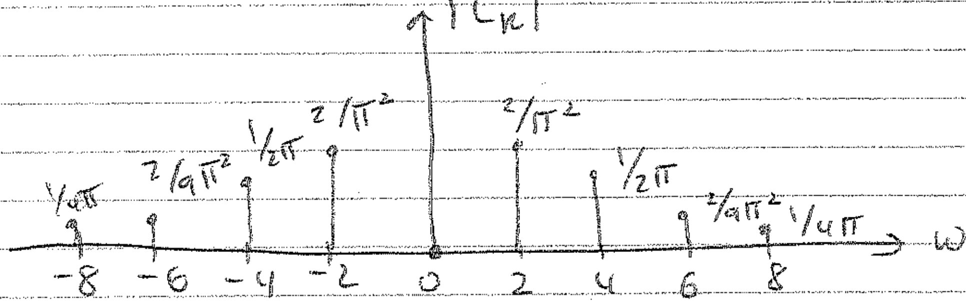
$$= \left[\frac{1}{-jk2\pi}\, e^{-jk\pi/2} + \frac{1}{\pi^2 k^2}\, e^{-j\pi/2 k} + \frac{-e^{jk\pi/2}}{jk2\pi} - \frac{e^{jk\pi/2}}{\pi^2 k^2}\right]$$

$$= -\frac{e^{jk\pi/2} - e^{-jk\pi/2}}{jk2\pi} + \frac{-1}{\pi^2 k^2}\left(e^{jk\pi/2} - e^{-jk\pi/2}\right)$$

$$= -\frac{\left(\frac{\pi}{2}k\right)}{\pi k j} - \frac{2j\sin\left(\frac{\pi}{2}k\right)}{\pi^2 k^2} = \frac{j\cos\left(\frac{\pi}{2}k\right)}{\pi k} - \frac{2j\sin\left(\frac{\pi}{2}k\right)}{\pi^2 k^2}$$

$=$

$C_1 = \dfrac{-2j}{\pi^2 k^2}, \quad C_2 = \dfrac{-j}{\pi k}, \quad C_3 = \dfrac{2j}{\pi^2 k^2}, \quad C_4 = \dfrac{j}{\pi k}$

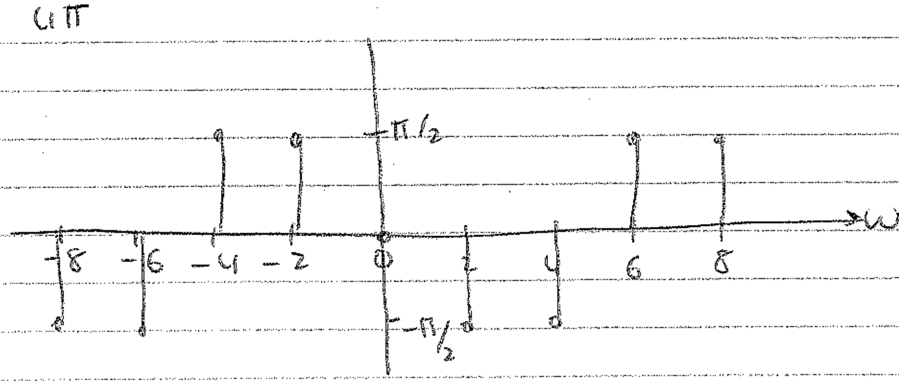



$|C_1| = \frac{2}{\pi^2 k^2} = \frac{2}{\pi^2}$

$|C_2| = \frac{1}{\pi 2}$

$|C_3| = \frac{2}{9\pi^2}$

$|C_4| = \frac{1}{4\pi}$



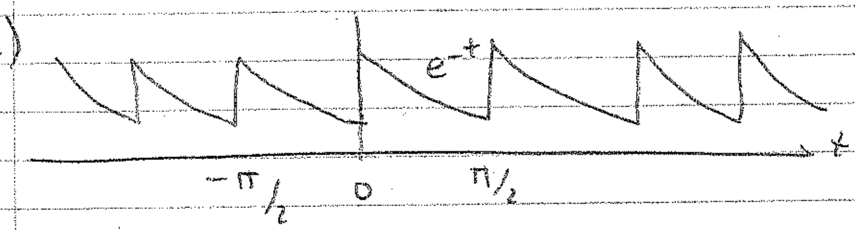

$\theta_1 = -\pi/2$

$\theta_2 = -\pi/2$ $\qquad A_k = 0 \rightarrow$ odd symmetric

$\theta_3 = \pi/2$

$\theta_4 = \pi/2$ $\qquad B_k = -2|C_k| \underbrace{\sin(\theta_k)}_{\pm\pi/2} \pm 1$

$= \pm 2|C_k|$

c) 


$T_0 = \pi/2 \qquad \omega_0 = \frac{2\pi}{\pi/2} = 4$

$$C_k = \frac{2}{\pi}\int_0^{\pi/2} e^{-t}\, e^{-jk4t}\, dt$$

$$= \frac{2}{\pi}\, \frac{e^{-t(1+j4k)}}{-(1+j4k)}\Bigg|_0^{\pi/2}$$

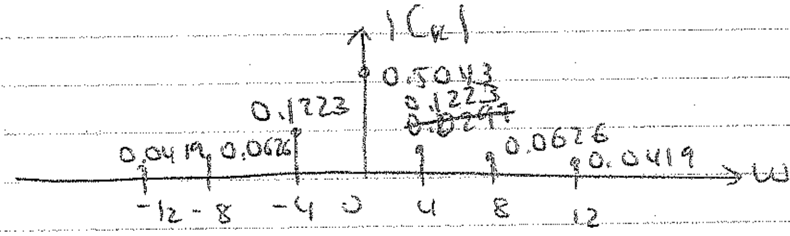
$$= \frac{2}{\pi}\left[\frac{e^{-\pi/2(1+j4k)}}{-(1+j4k)} + \frac{1}{1+j4k}\right]$$

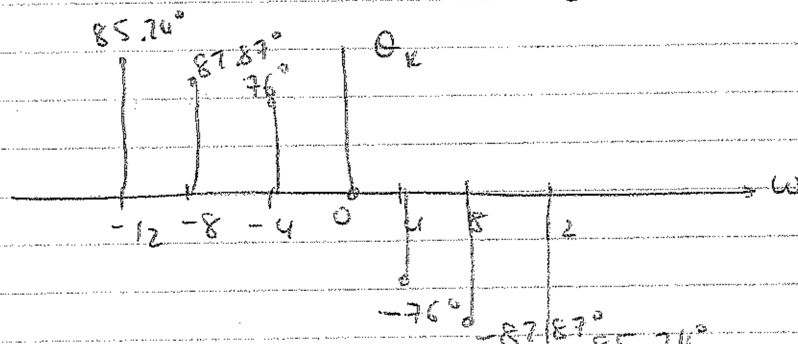
$$= \frac{2}{\pi}\left[\frac{e^{-\pi/2}\cdot e^{-j2\pi k}}{-(1+j4k)} + \frac{1}{1+j4k}\right]$$

$e^{-j2\pi k} = 1$

$$= \frac{2}{\pi}\left[\frac{1 - e^{-\pi/2}}{1+j4k}\right] = \frac{0.5043}{1+j4k}$$

$|C_k| = \frac{0.5043}{\sqrt{1+16k^2}} \qquad \theta_k = -\tan^{-1}(4k)$



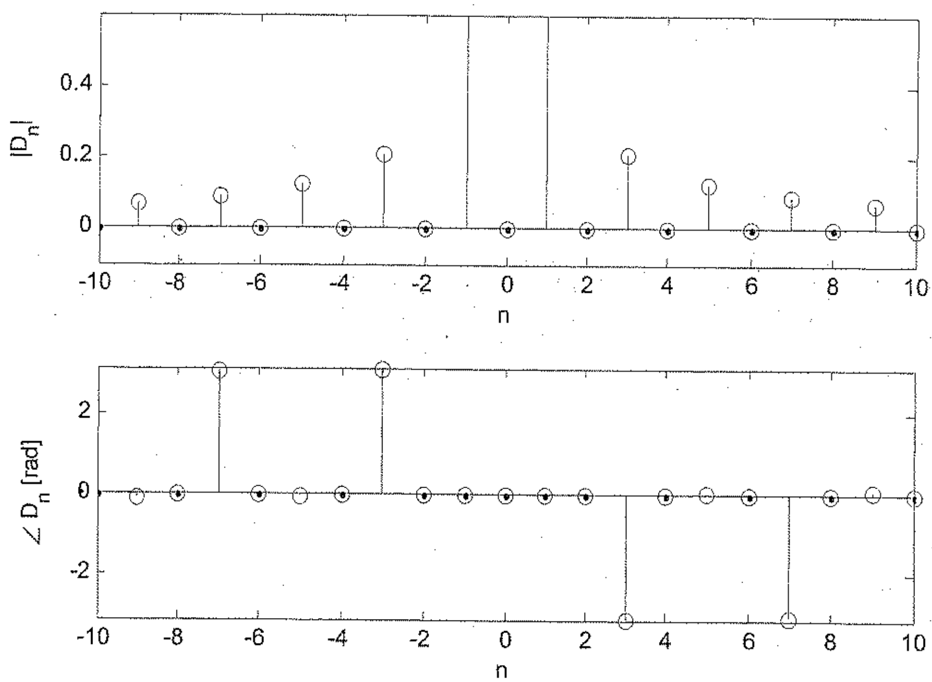

$A_k = \frac{1.0086}{1+16k^2}\cos(-\tan^{-1}(4k))$ $\qquad B_k = \frac{1.0086}{1+16k^2}\sin(-\tan^{-1}(4k))$

(5) For the signal in 2a
period = 4
take the signal from 0 to 4.
$x(1) = 1$ no need to redefine

$C_1 =$ 0.6366 agrees with our sketch.

See code

#5.




```
T_0=pi;N_0=256;T=T_0/N_0;t=(0:T:T*(N_0-1))';M=10;
x=[ones(1,64) -ones(1,128) ones(1,64)];
D_n=fft(x)/N_0;
n=[-N_0/2:N_0/2-1]';
clf;subplot(211);stem(n,abs(fftshift(D_n)),'k');
axis([-M M -0.1 0.6]);xlabel('n');ylabel('|D_n|');
subplot(212);stem(n,angle(fftshift(D_n)),'k');
axis([-M M -pi pi]);xlabel('n');ylabel('\angle D_n [rad]');
```